HANDWRITTEN

MATH NOES

Chapter No.8-17

9TH CLASS

Presented by:

Urdu Books Whatsapp Group

STUDY GROUP

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Chapter #8:- "Linear Graphs and their Applications."

Basic Concepts

i) Coordinate Plane.

ii) Ordered Pair.

iii) Coordinate Axes.

iv) Abbcissa.

v) Ordinate.

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vi) Origin.

vii) Quardrant.

viii) Collinear Points

ix) Ex. 8.1.

x) Ex. 8.2, Q3 (only)

xi) Conversion.

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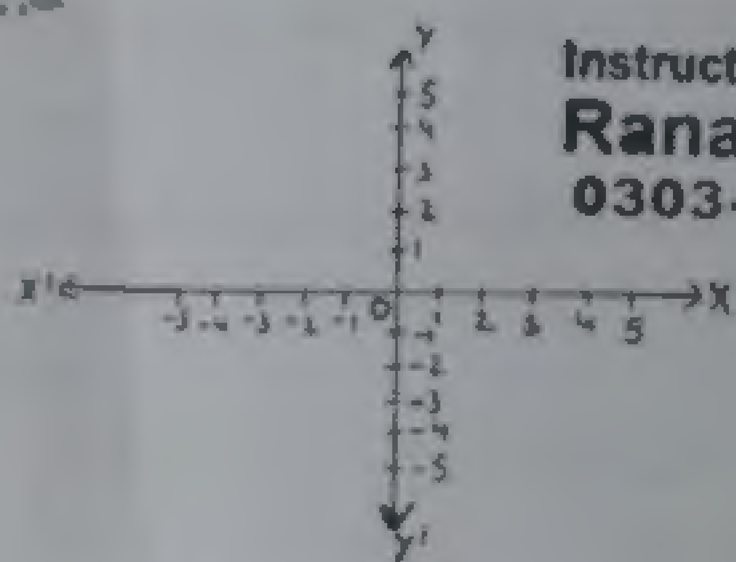
xii) Note.

xiii) Review Ex 8. (without Q6.).

i) Coordinate Plane:-

The plane formed by two straight lines perpendicular to each other is called coordinate plane and the lines are called coordinate axes.

Example:-



ii) Ordered Pair:-

An ordered pair is a pair of elements in which elements are written in specific order.

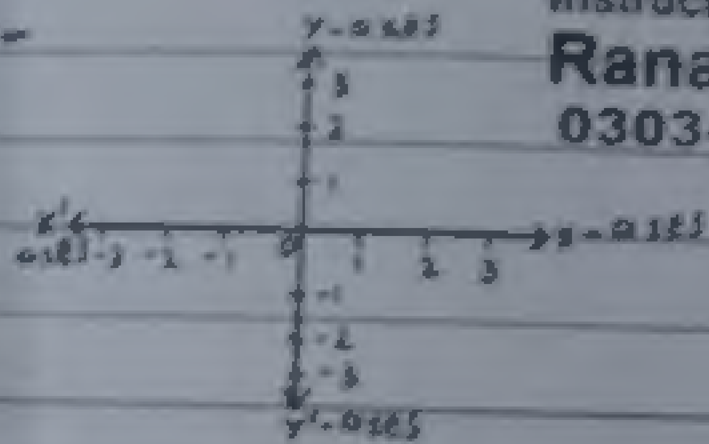
Example:-

(x, y), $(0, -1)$, etc.

iii) Coordinate axes:-

The plane formed by two straight lines perpendicular to each other is called coordinate plane and the lines are called coordinate axes.

Example:-



iv) Abbcissa:-

The x-coordinate of a point is called abbcissa.

Example:-

(3,4)

Here, 3 is a abbcissa.

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v) Ordinate:-

The y coordinate of a point is called ordinate.

Example:-

(3,4)

Here, 4 is a ordinate.

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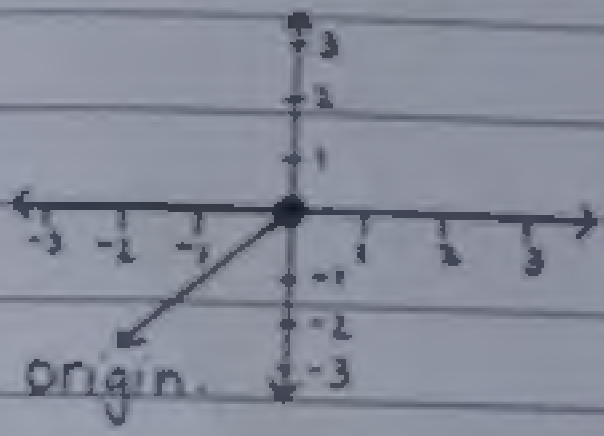
vi) Origin:-

The point of intersection of two coordinate axes is called origin. It is represented by "O"

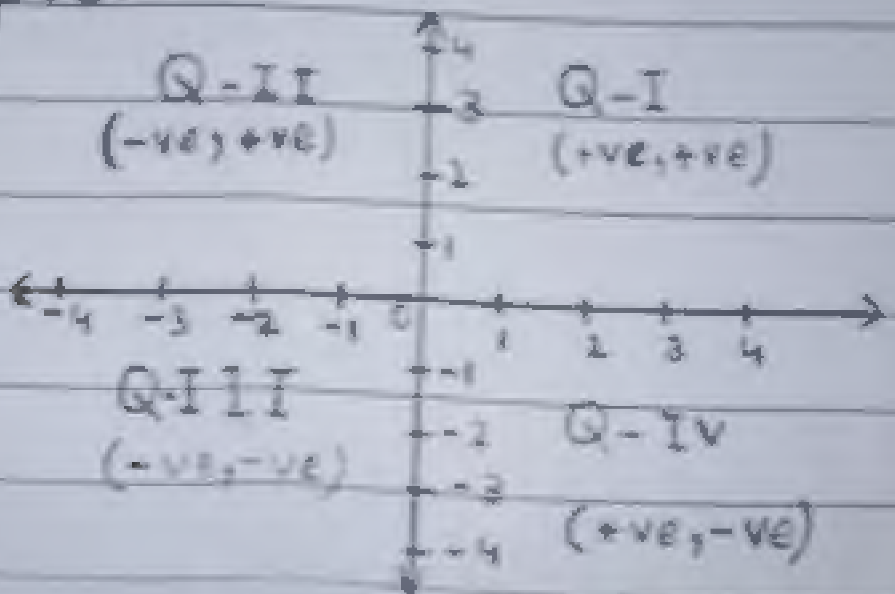
Example:-



vii) Quardrant:-

In any one of quardrants of the plane namely XOY, YOX', X'OY' and Y'OX respectively called first, 2nd, 3rd and 4th quardrant of the plane subdivided by the coordinate axes of the plane. They are denoted by Q-I, Q-II, Q-III, Q-IV respectively.

Example:-



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viii) Collinear Points:-

The set of points which lie on the same line are called collinear points.

Example:-

Here, A and B and C are collinear Points.

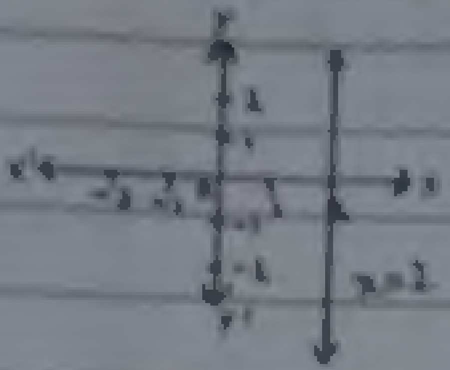
ix) Ex 8.1:-

1- Determine the quardrant of the coordinate plane in which the following points lie;

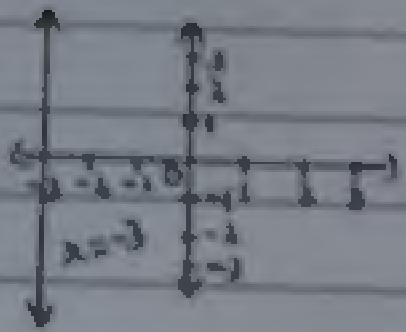
P(-4,3)
This point is lie on the Q-II.
Q(-5,-2)
This point is lie on the Q-III.
R(2,2)
This point is lie On the Q-I.
S(2,-6)
This point is lie on the Q-IV.

2- Draw the graph of each of the following.

a) $x = 2$

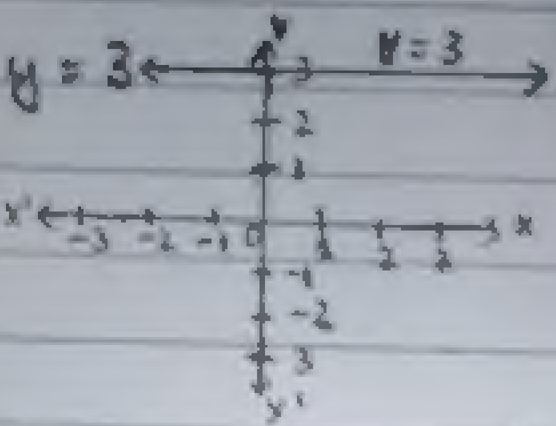


b) $x = -3$

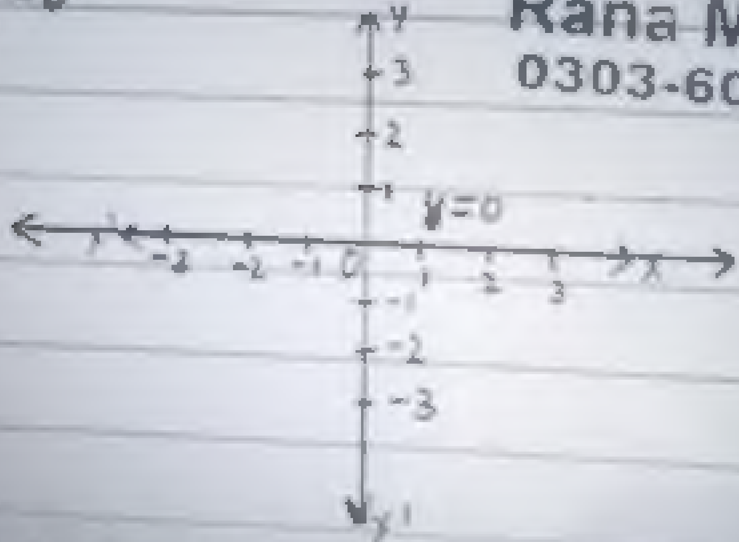


c) $y = -1$

d) $y = 3$



e) $y = 0$



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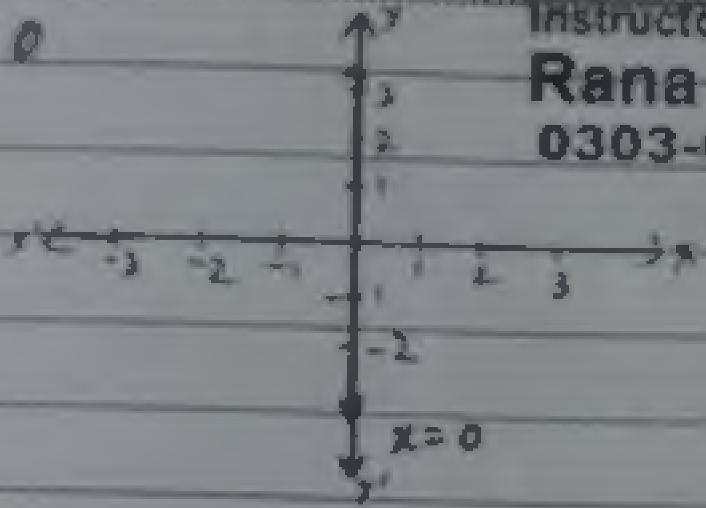
f) $x = 0$

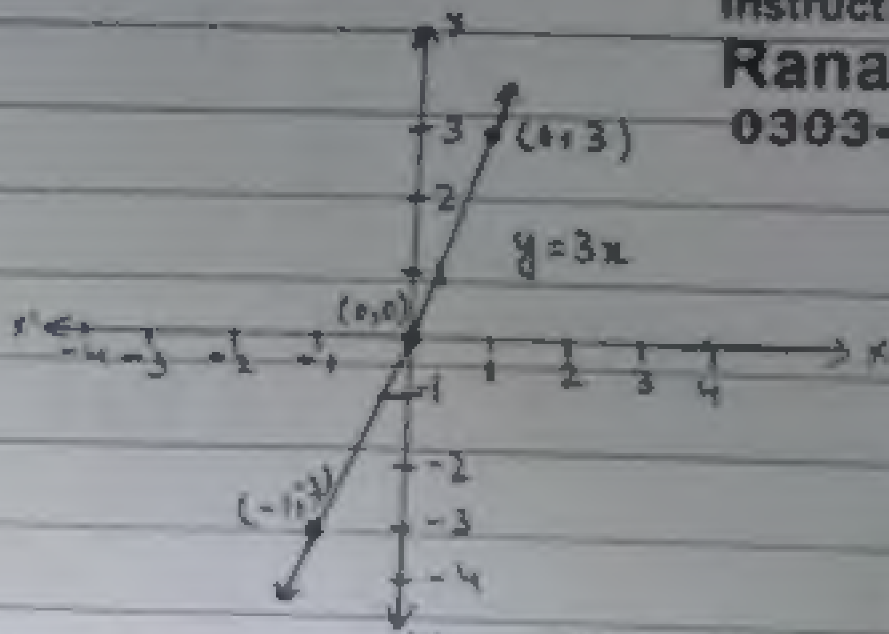


g) $y = 3x$

$y = 3x \longrightarrow ①$

Table:-

x	1	-1	0
y	3	3	0

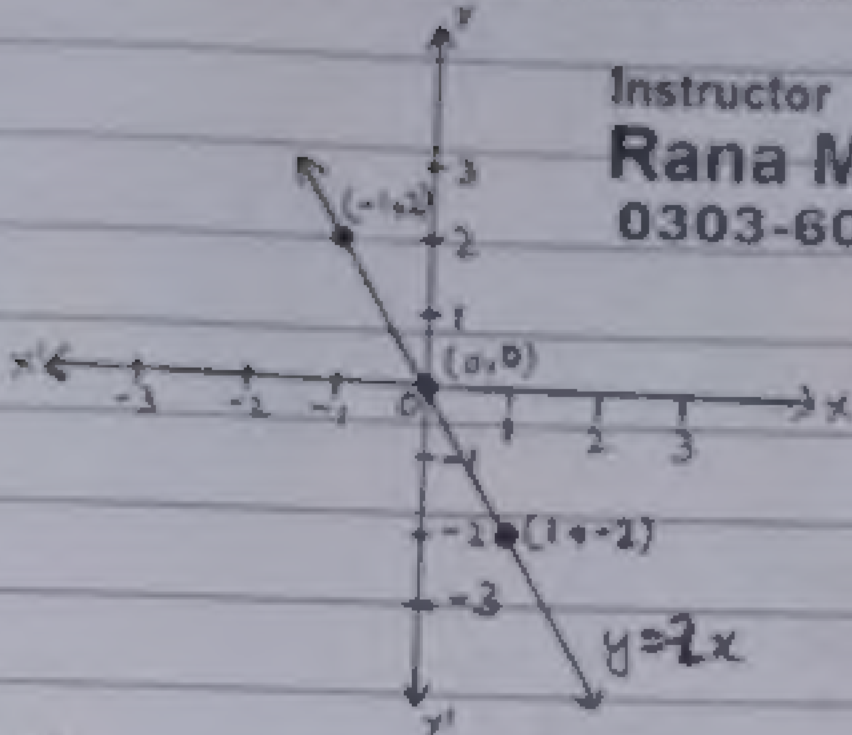


h) $-y = 2x$

$y = -2x \longrightarrow ①$

Table:-

x	1	-1	0
y	-2	2	0



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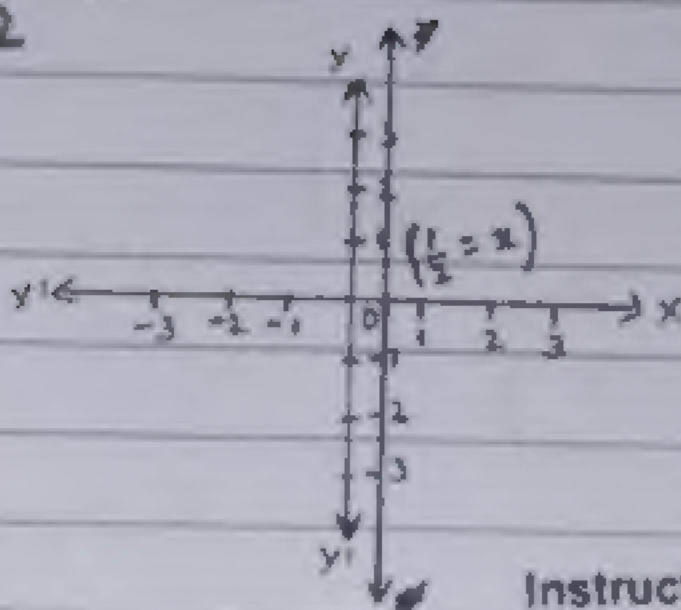
i) $\frac{1}{2} = x$



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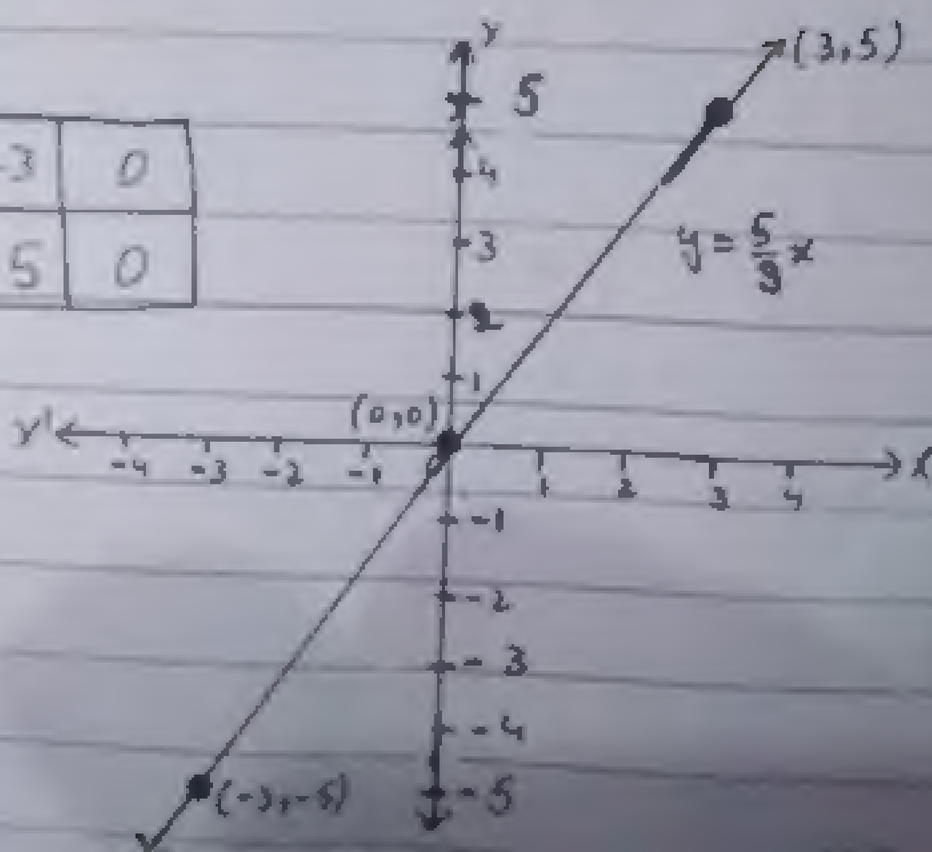
j) $3y = 5x$

$y = \frac{5}{3}x$

Table:-

x	3	-3	0
y	5	-5	0

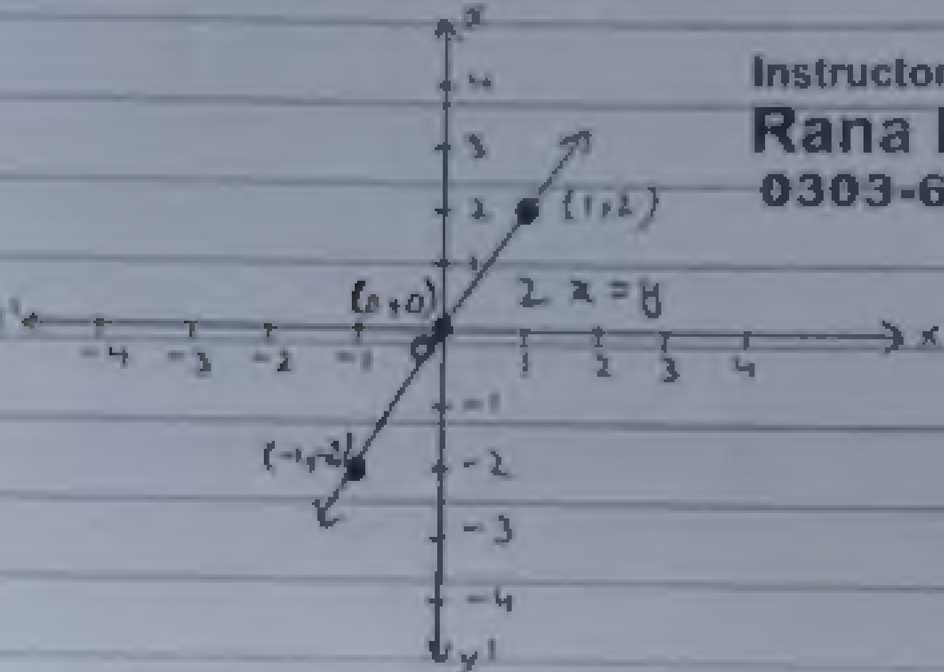


k) $2x - y = 0$

$2x = y \longrightarrow (1)$

Table:-

x	1	-1	0
y	2	2	0



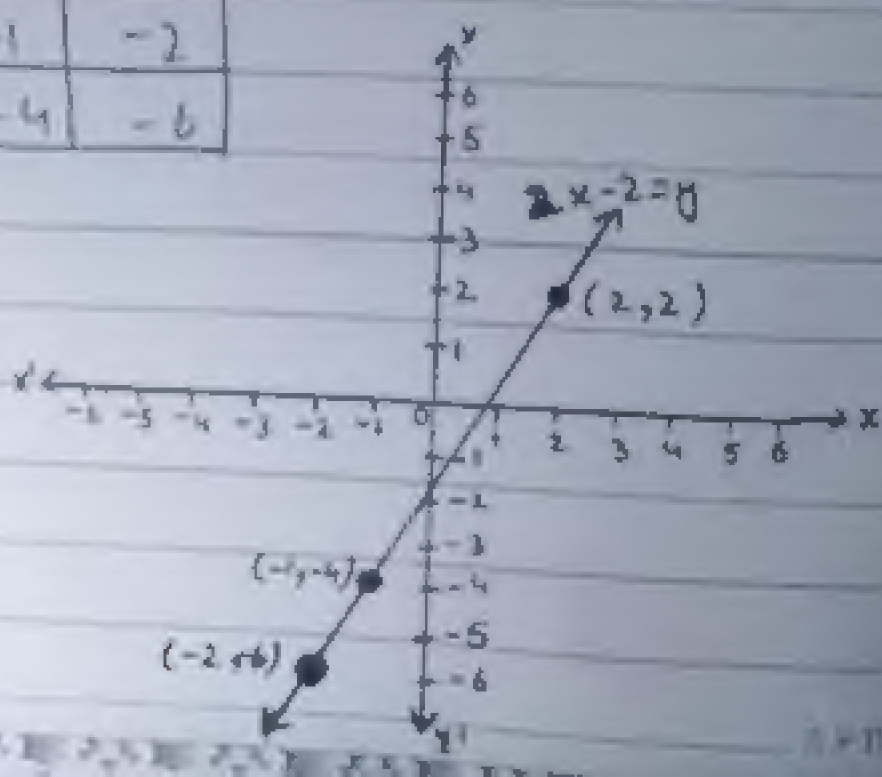
l) $2x - y = 2$

$2x = 2 + y$

$2x - 2 = y \longrightarrow (1)$

Table:-

x	2	-1	-2
y	2	-4	-6



m) $x - 3y + 1 = 0$

$x + 1 = 3y$

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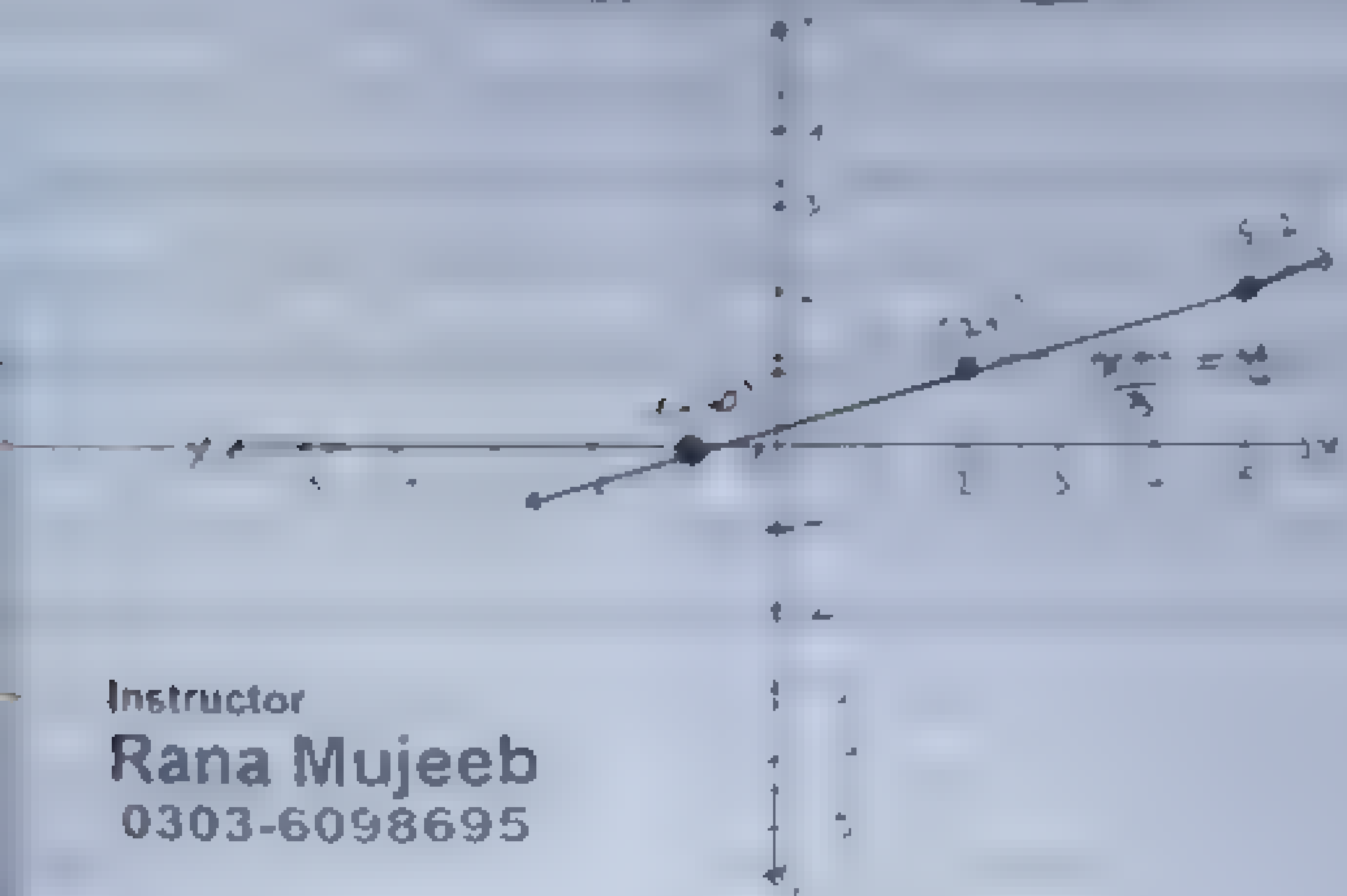

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n) $3x - 2y + 1 = 0$

$3x + 1 = 2y$

$\frac{3x+1}{2} = y$

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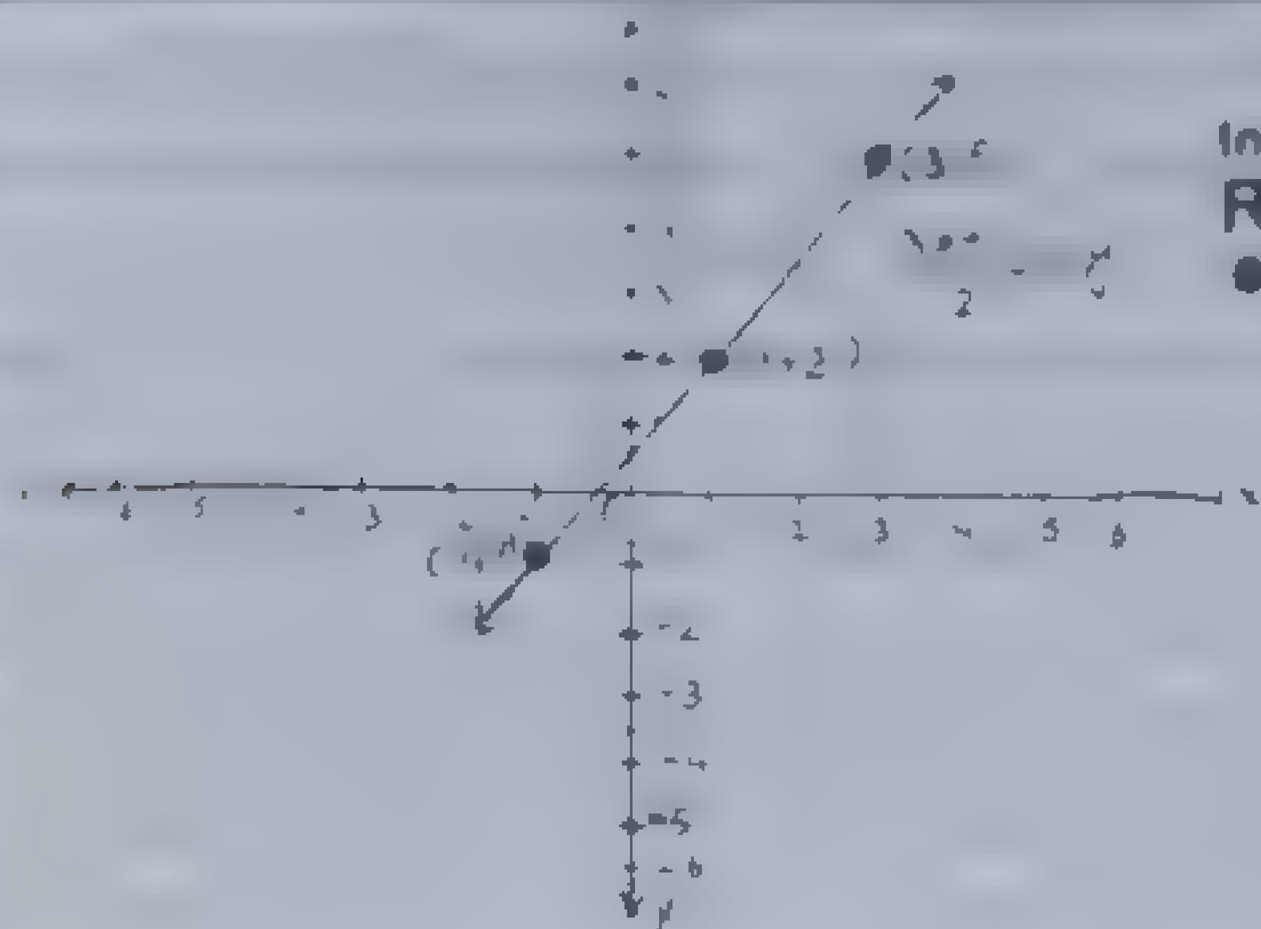
Table:

[illegible]	[illegible]	[illegible]	[illegible]
1	2	-1	5



3- Are the following lines (i) parallel to x-axis (ii) parallel to y-axis.

i) $2x-1=3$

$2x = 3+1$

$2x = 4$

$x = \frac{4}{2}$

$x = 2$

This line is parallel to y-axis

ii) $x+2=1$

$x = 1-2$

$x = -1$

This line is parallel to y-axis.

iii) $2y + 3 = 2$

This line is parallel to x-axis.

iv) $x + y = 0$

$x = -y$

This line is neither parallel to x-axis and nor parallell to y-axis.

V) $2x - 2y = 0$

$2x = 2y$

$x = \frac{2y}{2}$

$x = y$

This line is neither parallel to x-axis and nor parallel to y axis.

4- Find the value of m and c of the following lines by expressing them in the form $y = mx + c$.

a) $2x + 3y - 1 = 0$

Let.

$y = mx + c \rightarrow (1)$

Now,

$2x + 3y - 1 = 0$

$3y = 1 - 2x$

$y = \frac{1 - 2x}{3} \rightarrow (2)$

b) $x - 2y = -2$

c) $3x + y$

d) $2x - y = 7$

Let,

$y = mx + c \rightarrow (1)$

Given

$2x - y = 7$

$2x - 7 = y \rightarrow (2)$

By comparing eq (1) & (2)

$\boxed{m = 2, \; c = -7}$

e) $3 - 2x + y = 0$

Let,

$y = mx + c \rightarrow (1)$

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Given

$3 - 2x + y = 0$

$y = -3 + 2x \rightarrow (2)$

By comparing eq (1) & (2)

$\boxed{m = 2, \; c = -3}$

f) $2x = y + 3$

Let,

$y = mx + c \rightarrow (1)$

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Given

$2x = y + 3$

$2x - 3 = y \rightarrow (2)$

By comparing eq (1) & (2)

$\boxed{m = 2, \; c = -3}$

5- Verify whether the following point lies on the line $2x-y+1=0$ or not.

i) $(2,3)$

Let,

$2x-y+1=0 \rightarrow (i)$

Put $(2,3)$ in eq (i)

$2(2)-3+1=0$

$4-3+1=0$

$1+1=0$

$2\neq 0$

The point does not lie on the line

ii) $(0,0)$

Let,

$2x-y+1=0 \rightarrow (i)$

Put $(0,0)$ in eq (i)

$2(0)-0+1=0$

$0-0+1=0$

$1\neq 0$

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The point does not lie on the line

iii) $(-1,1)$

Let,

$2x-y+1=0 \rightarrow (i)$

Put $(-1,1)$ in eq (i)

$2(-1)-1+1=0$

iv) (2, 5)

Let,

$2x - y + 1 = 0 \rightarrow (i)$

Put (2, 5) in eq (i):

$2(2) - 5 + 1 = 0$

$4 - 5 + 1 = 0$

$-1 + 1 = 0$

$0 = 0$

The point lie on the line

v) (5, 3)

Let,

$2x - y + 1 = 0 \rightarrow (i)$

Put (5, 3) in eq (i):

$2(5) - 3 + 1 = 0$

$10 - 3 + 1 = 0$

$8 \neq 0$

The point does not lie on the line.

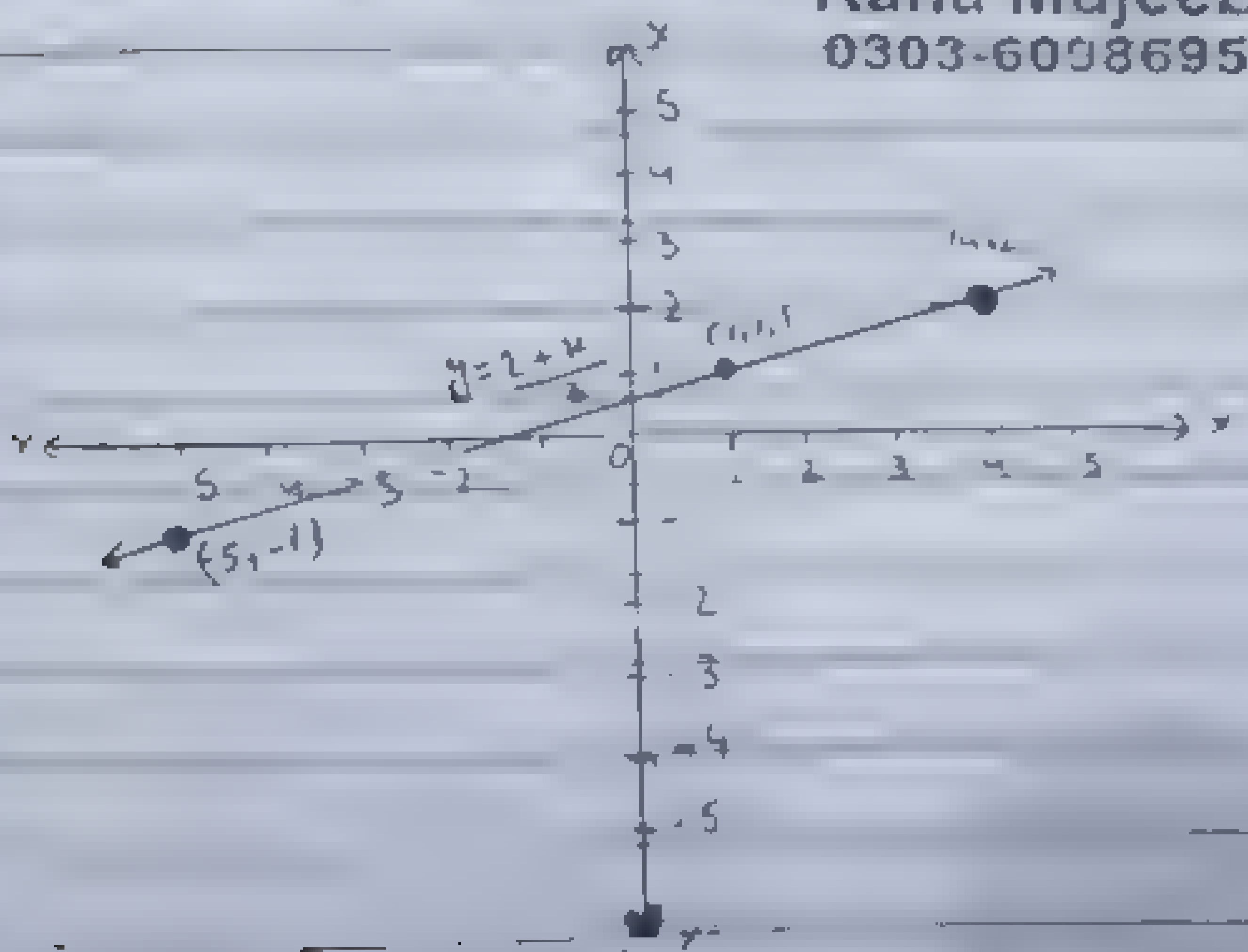
Ex: 8.2

Q3: Sketch the graph on graph paper of the following.

(V) $x - 3y + 2 = 0$

$y = \frac{2+x}{3}$

x	1	4	-5
y	1	2	-1

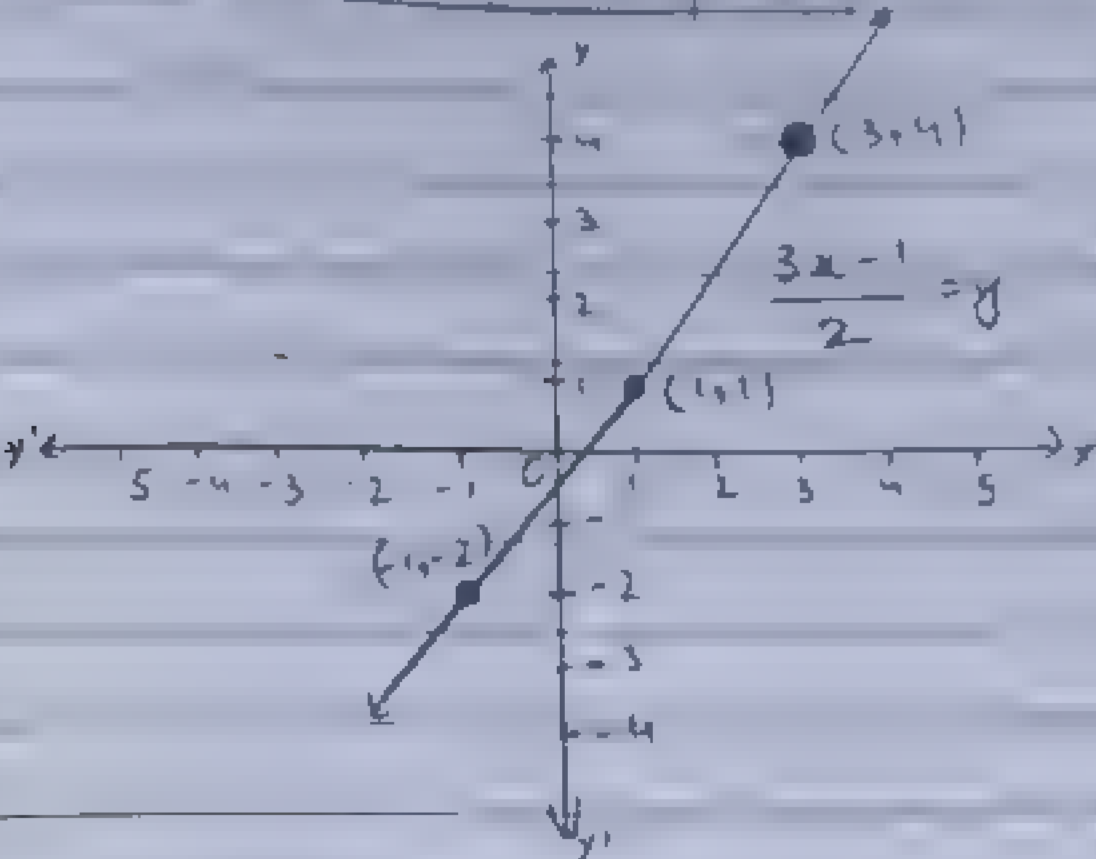


b) $3x - 2y - 1 = 0$

$3x - 1 = 2y$

$\frac{3x-1}{2} = y$

x	1	-1	3
y	1	-2	4



c) $2y - x + 2 = 0$

$2y = x - 2$

$y = \frac{x-2}{2}$

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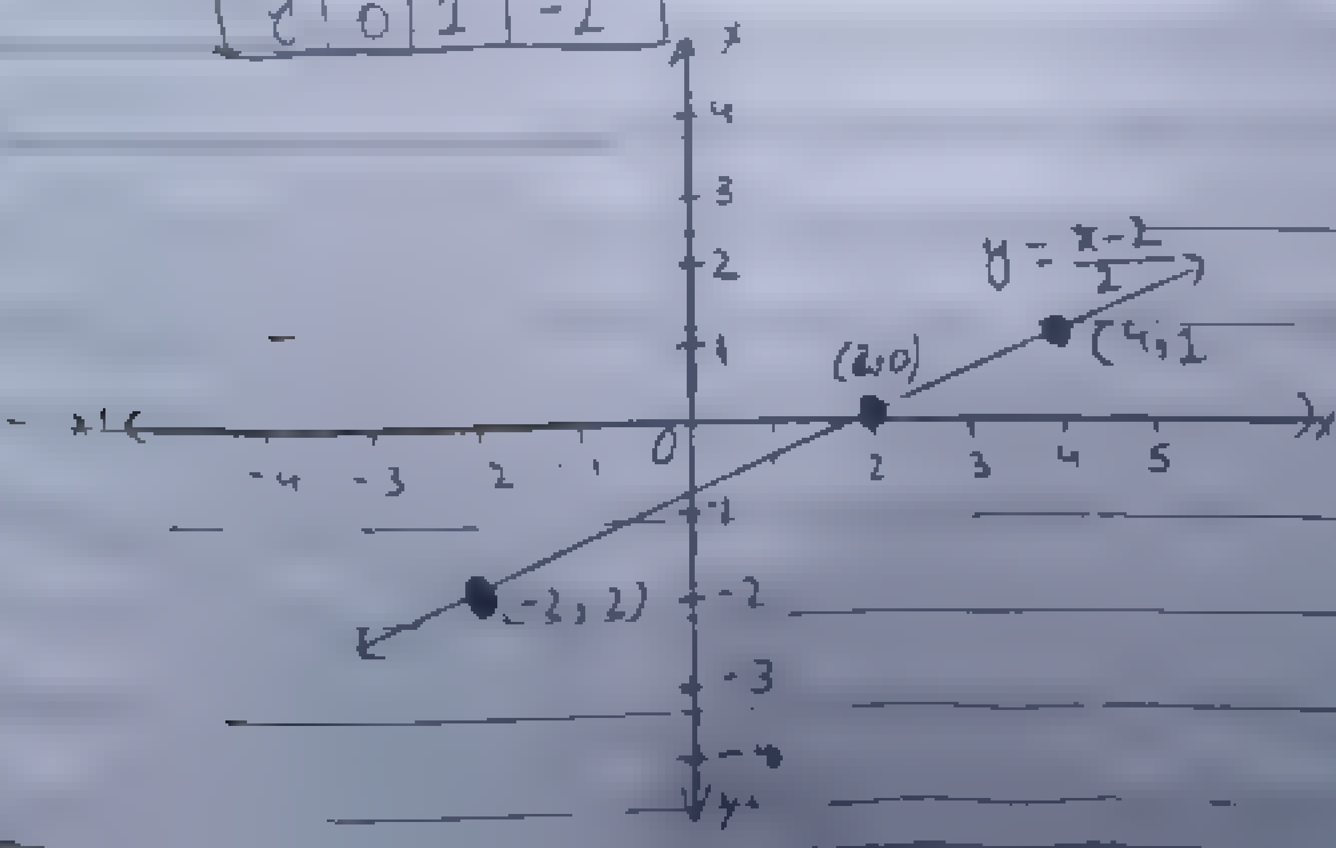
Table

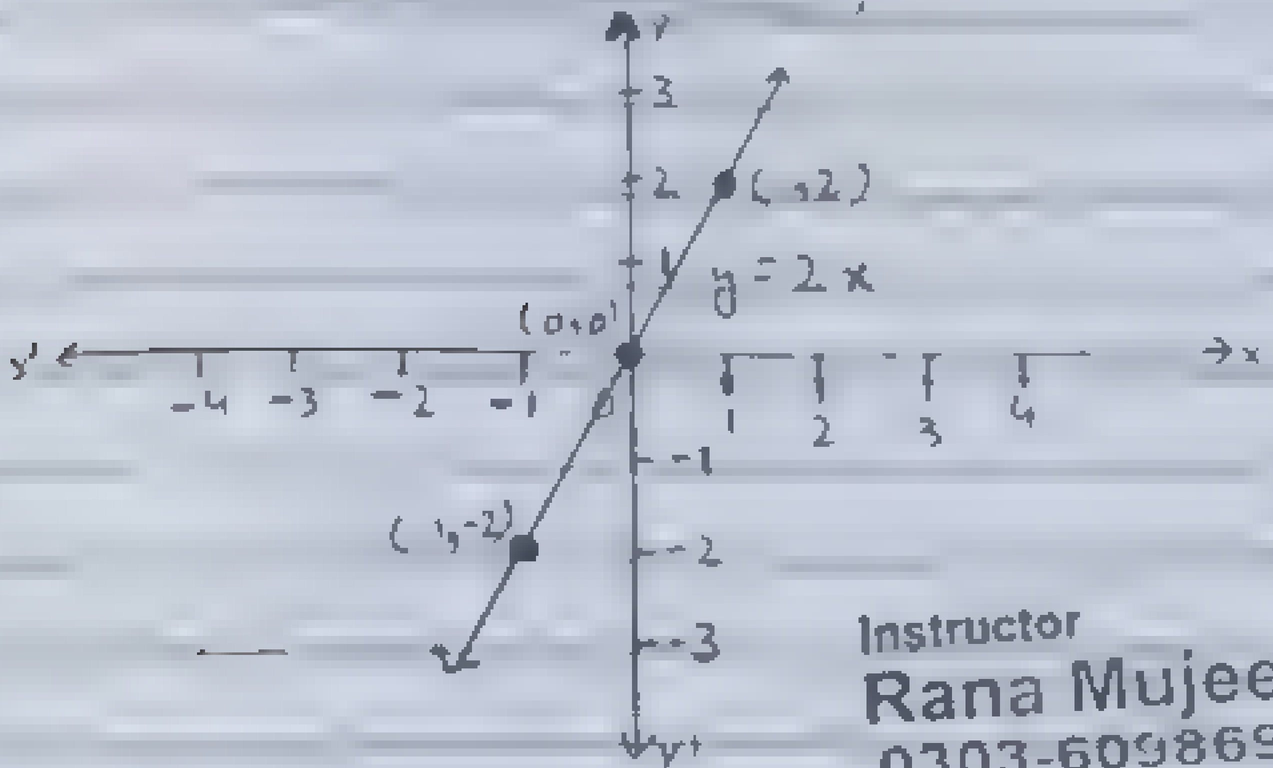
x	2	4	-2
y	0	1	-2



d) $y - 2x = 0$

Table:–

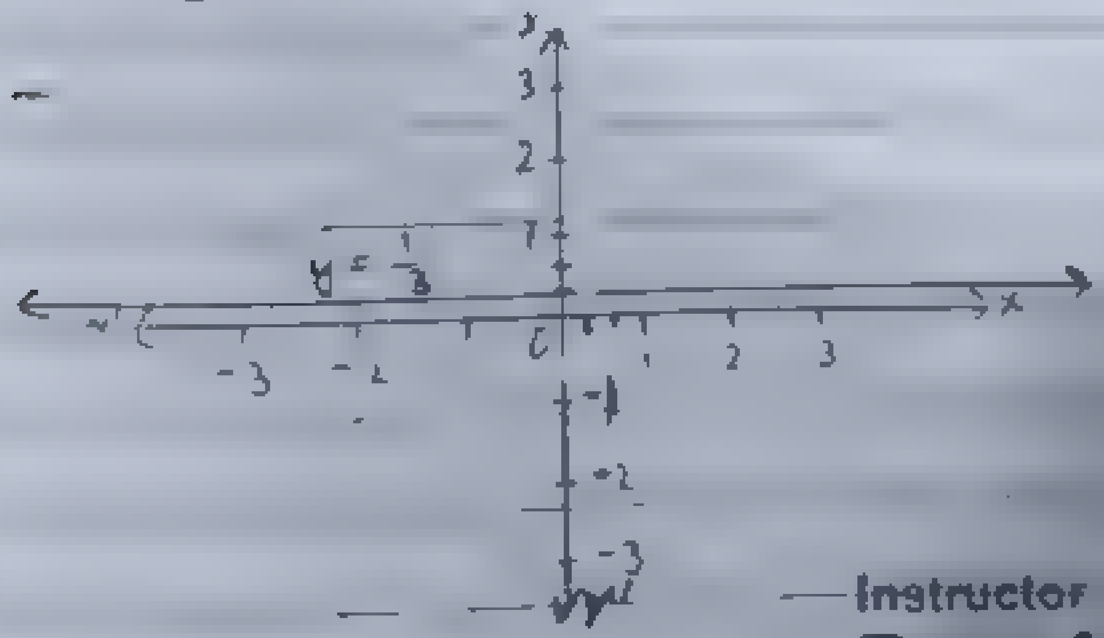
x	1	-1	0
y	2	-2	0



e) $3y - 1 = 0$

$3y = 1$

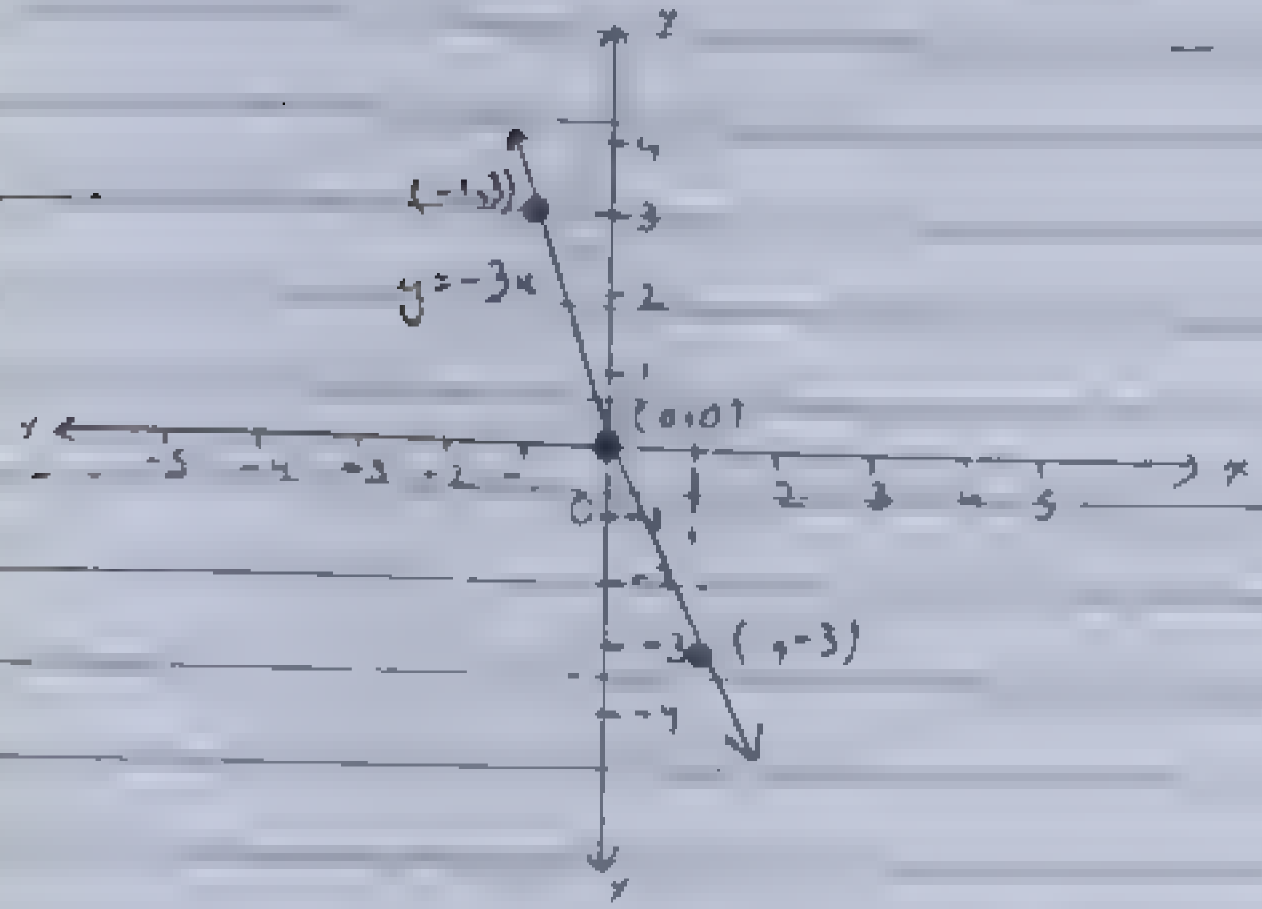
$y = \frac{1}{3}$



f) $y + 3x = 0$

$y = -3x$

x	1	-1	0
y	-3	3	0



g)

$2x + 6 = 0$

$2x = -6$

$x = -\frac{6}{2}$

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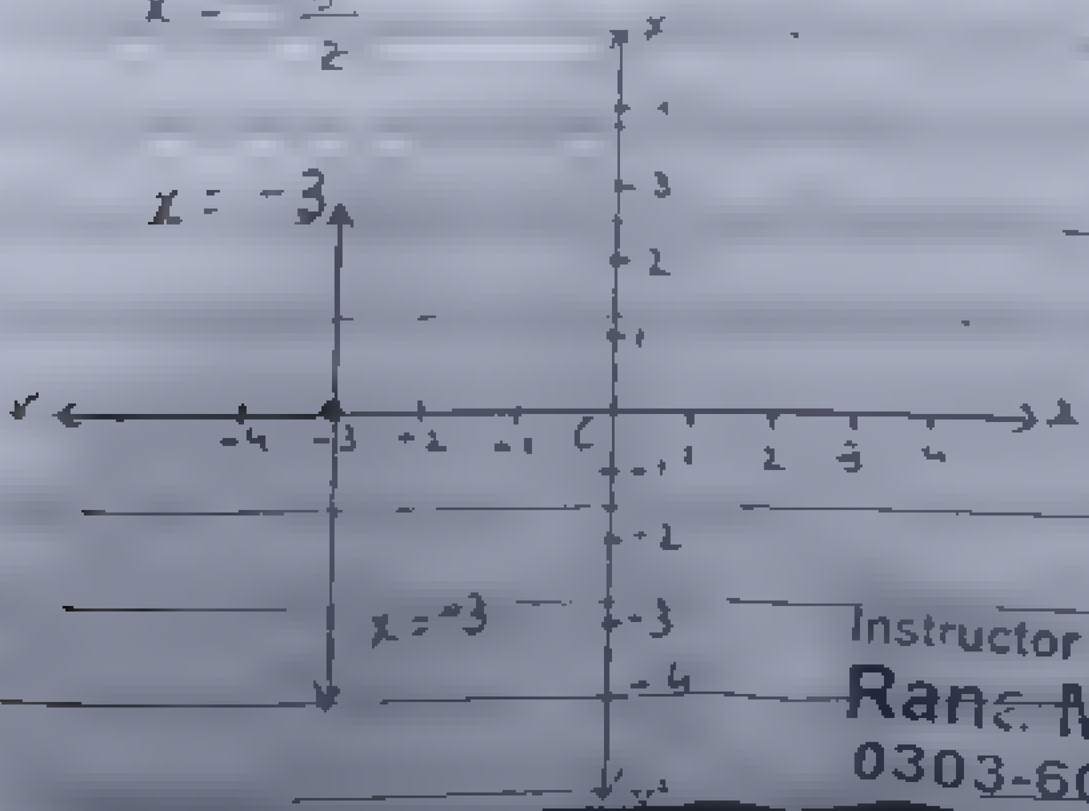

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1) Conversion.-

a) Km to mile / mile to km.-

[illegible]

1 mile = 1.6 km

b) Hectare to Acre / Acre to Hectare.-

1 hectare = 2.5 acre

1 acre = 0.4 hectare

c) US Dollar to PKR :-

1 US \$ = 166 Rupees

d) C to F / F to C:-

$C = \frac{5}{9}(F-32)$

$F = \frac{9}{5}C + 32$

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iii) Note:-

- (x, y) is an ordered pair in which 1st element is x and 2nd element is y such that $(x,y) \neq (y,x)$ where, $x \neq y$.
- (2,3) and (3,2) are two different ordered pair.
- $(x,y) = (m,n)$ only if $x = m$ and $y = n$
- Each point P of plane can be identified by the coordinates of the pair (x,y) and is represented by P(x,y).
- All the points of plane may y-coordinate y = 0 if they lie on x-axis i.e., P(2,0) lies on x-axis.

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the [illegible] 0303 6098695

[illegible]

Review Ex 8:-

1- Choose the correct options.

(i) [illegible]

(a) [illegible] (b) (,1) (c) (1,1) (d) (,1)

(ii) If $(x,0)=(0,y)$, then (x,y) is,

(a) [illegible] (b) (1,0) (c) (0,0) (d) (1,1)

(iii) Point [illegible] lies in quadrant,

(a) I (b) II (c) III (d) IV

(iv) If $y = 2x+1$, $x = 2$ then y is,

(a) 2 (b) 3 (c) 4 (d) 5

(v) Which ordered pair satisfies the equation $y = 2x$?

(a) (,2) (b) (2,1) (c) (2,2) (d) (0,1)

2- Identify the following which statement as true or false?

(i) The point O(0,0) is in quadrant III. False.

(ii) The point P(2,0) lie on x-axis. True.

(iii) The graph of $x=-2$ is a verticle line. True.

(iv) $y=2$ is a horizontal line. True.

(v) The point Q(-1,2) is in quadrant III. False.

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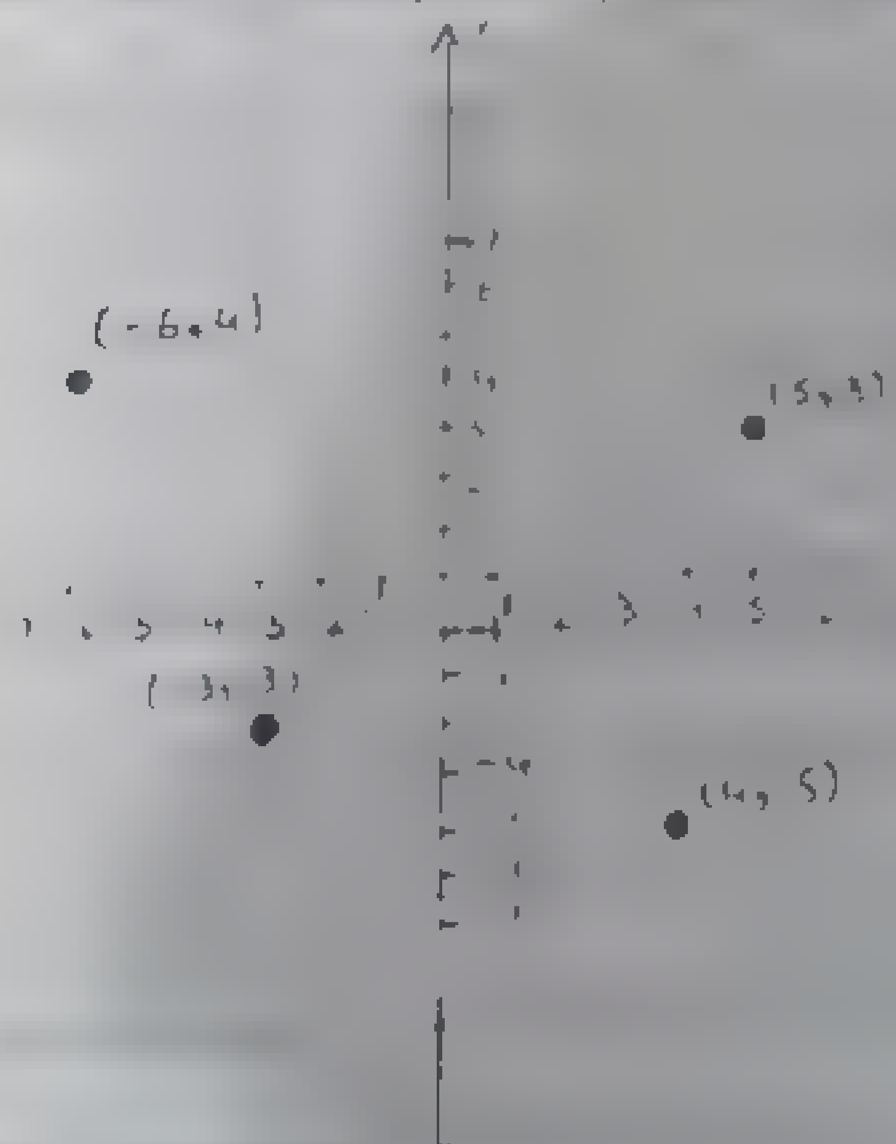
false.

False.

false.

3- Draw the following graph on graph paper.

$(-3,-3), (-6,4), (4,5), (5,3)$



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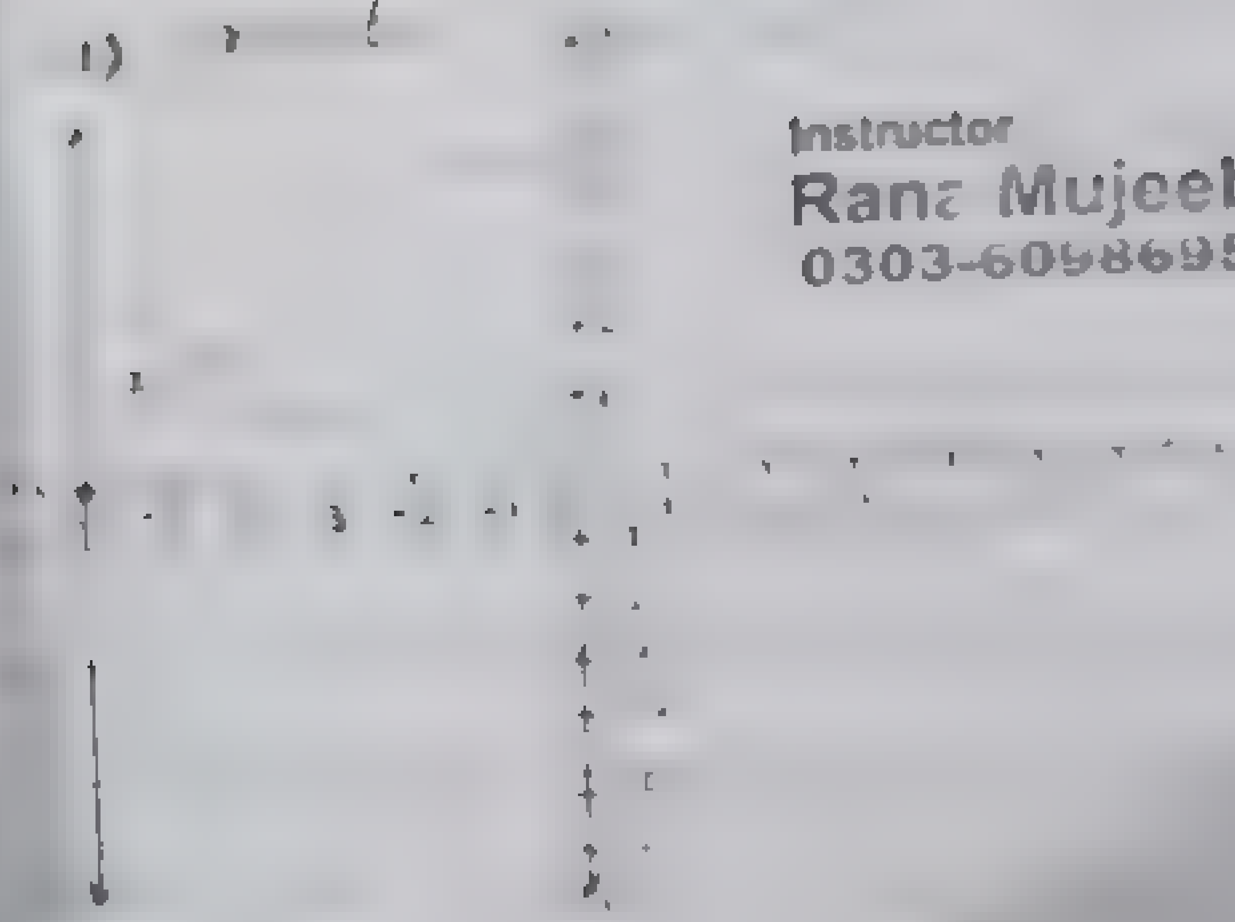
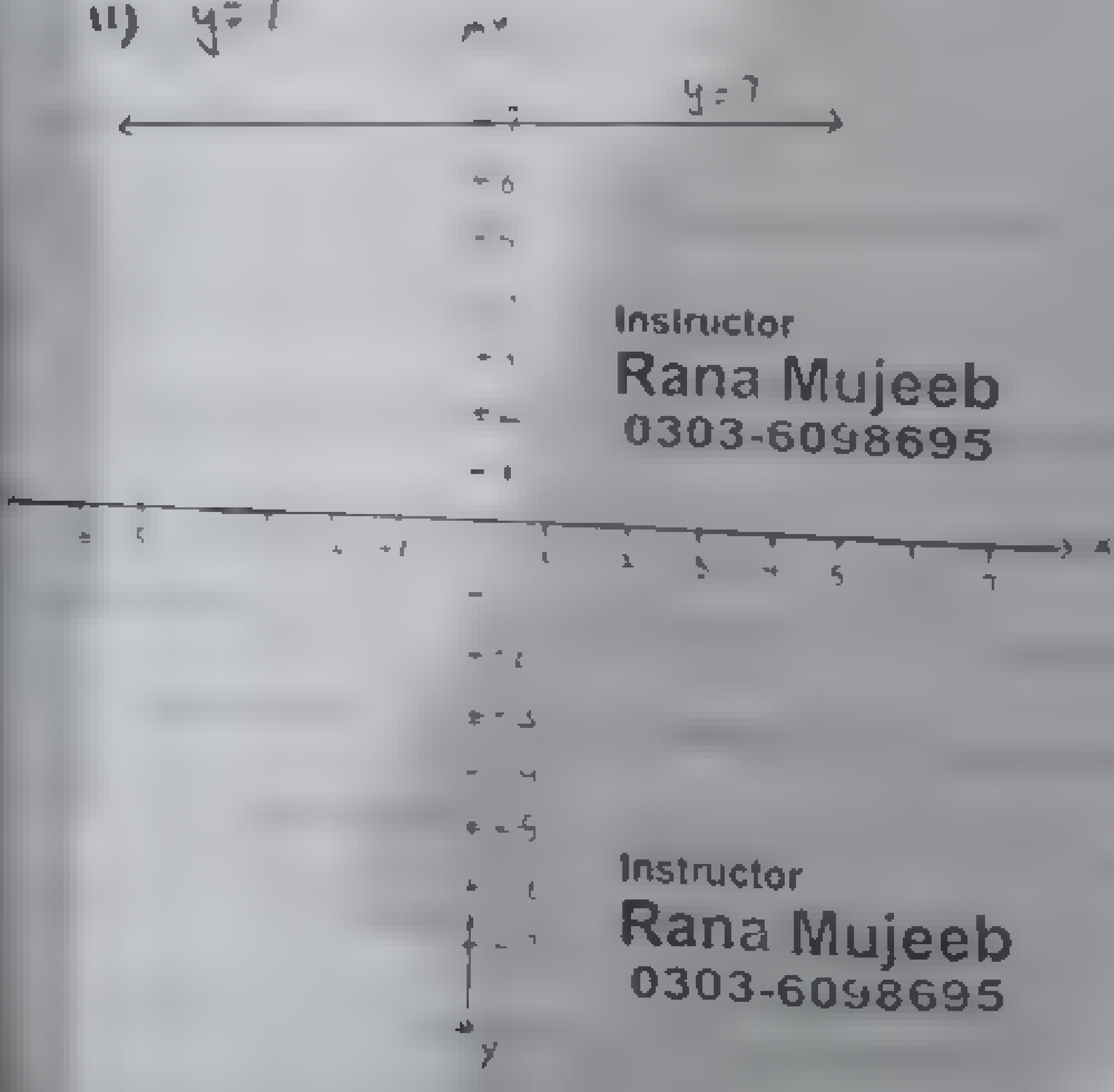
4- ... the graph of the following.

i)



ii) $y = 7$



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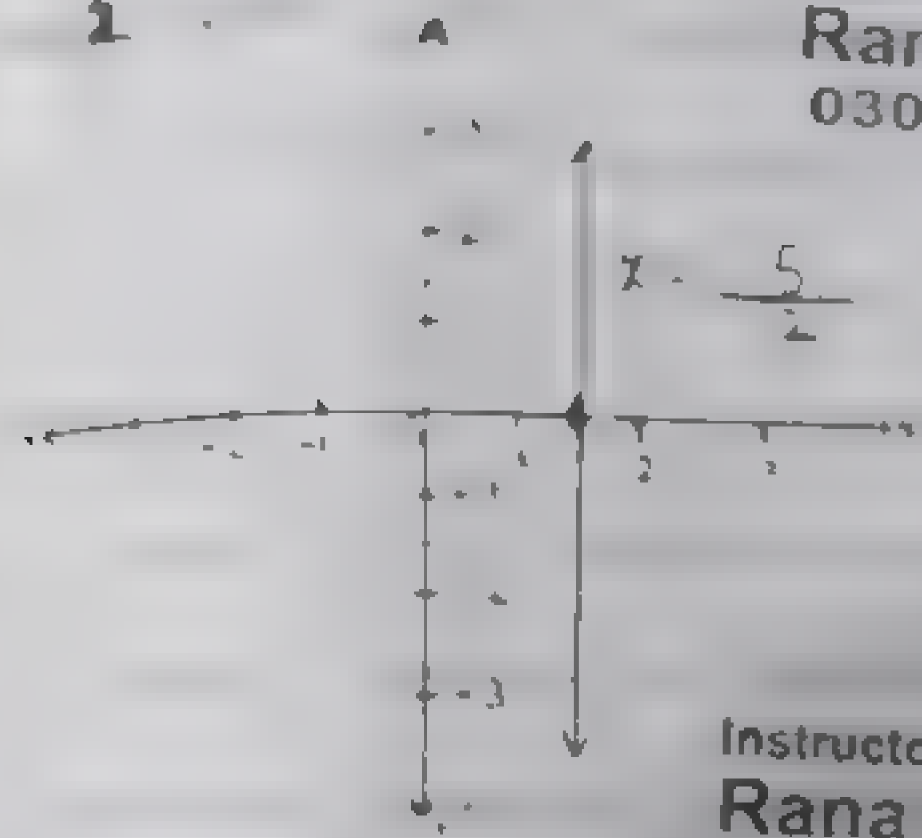
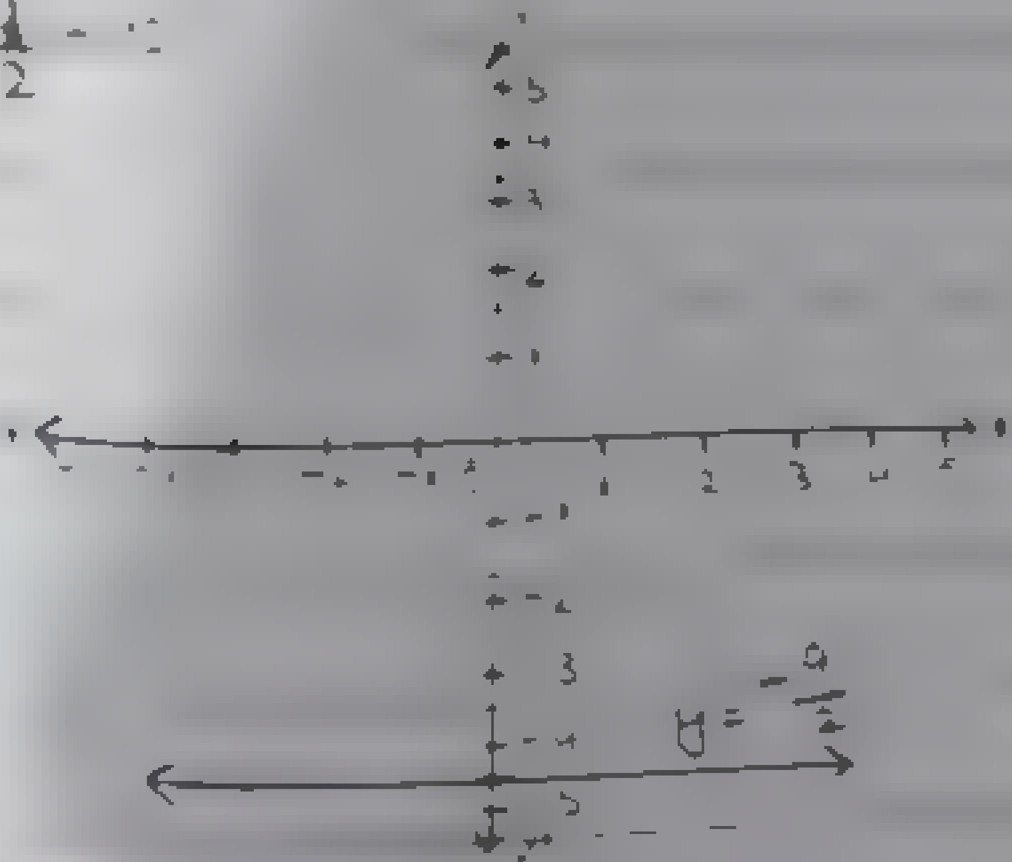
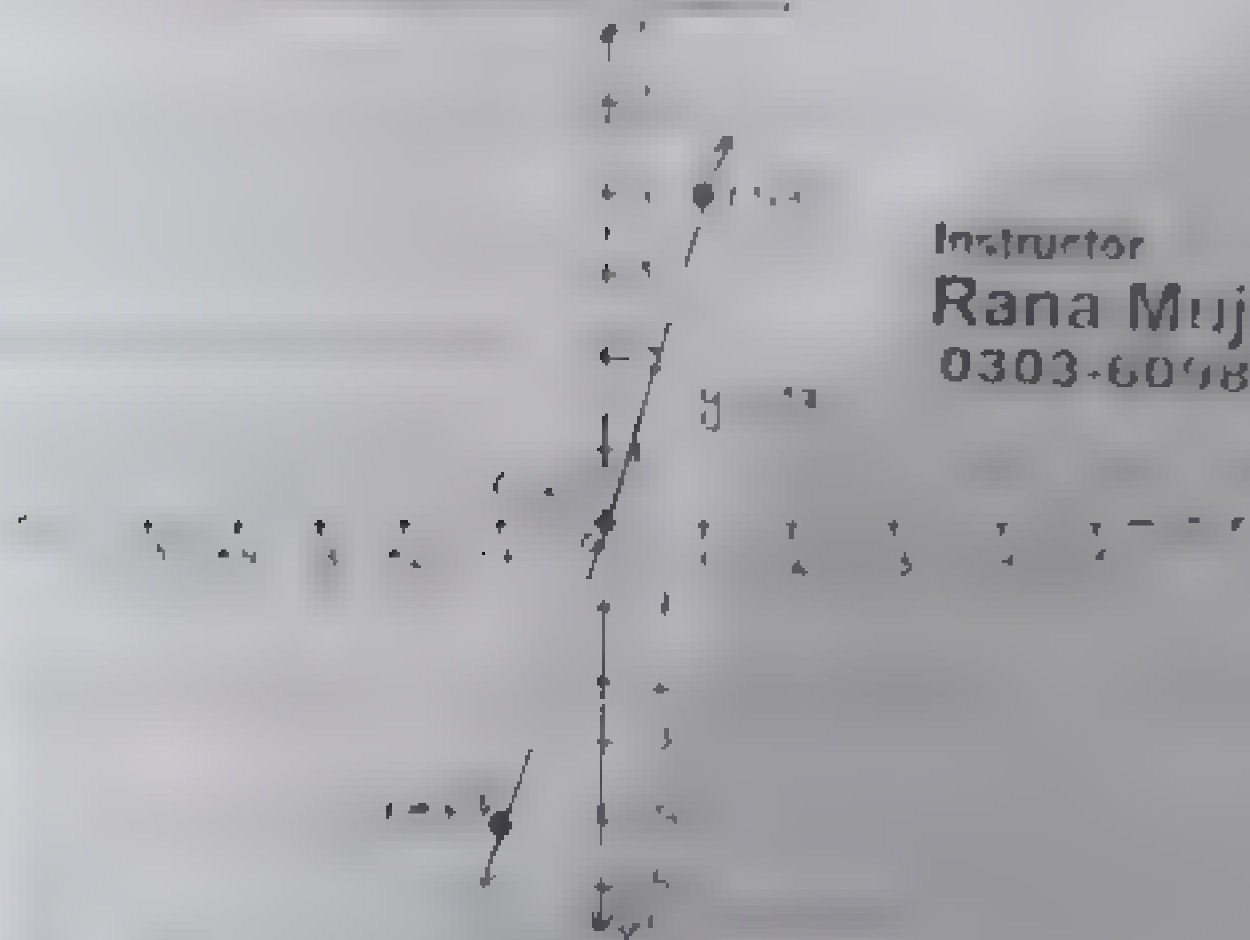


iv) $y = -\frac{9}{2}$





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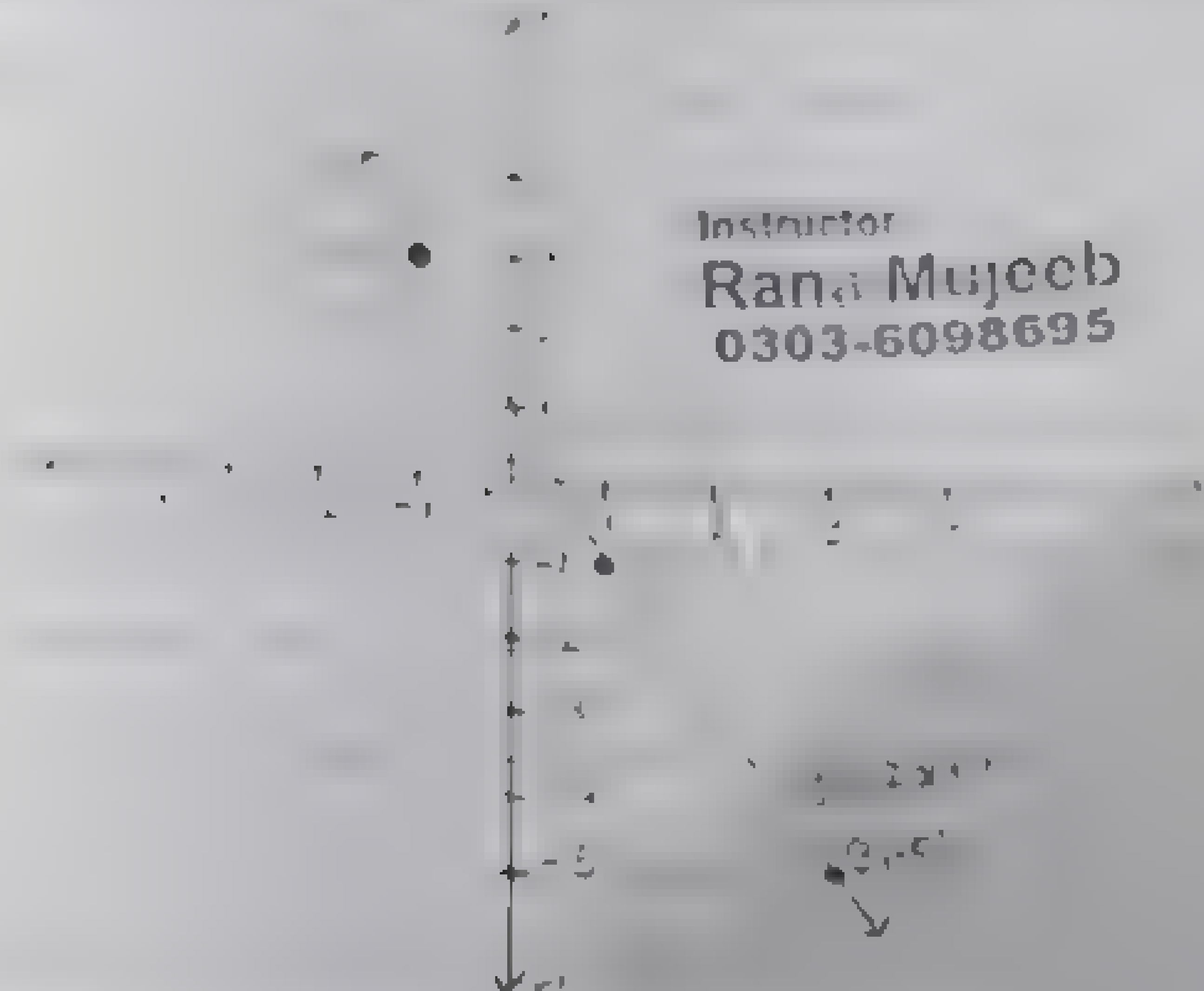
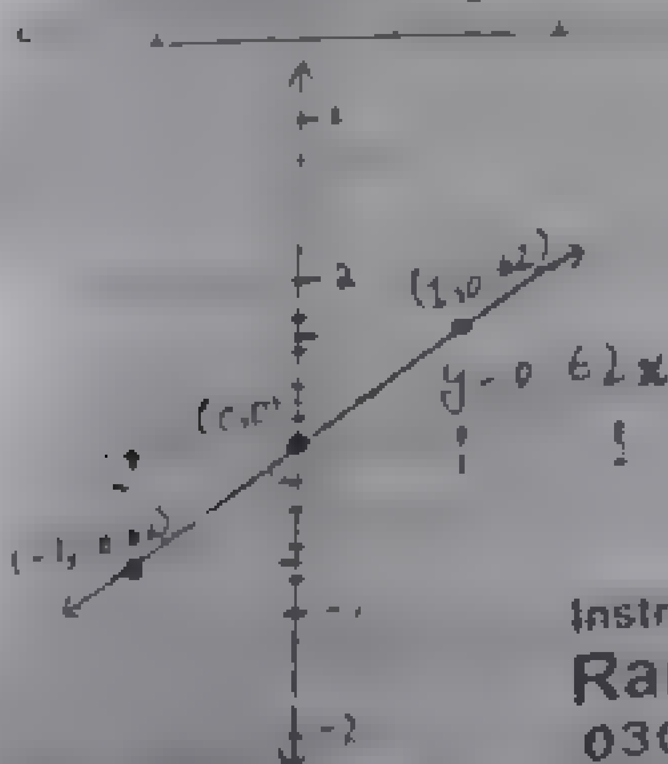
x	1	-1	3
y	-1	3	-5



5- Draw the following graph

i) $y = 0.62x$

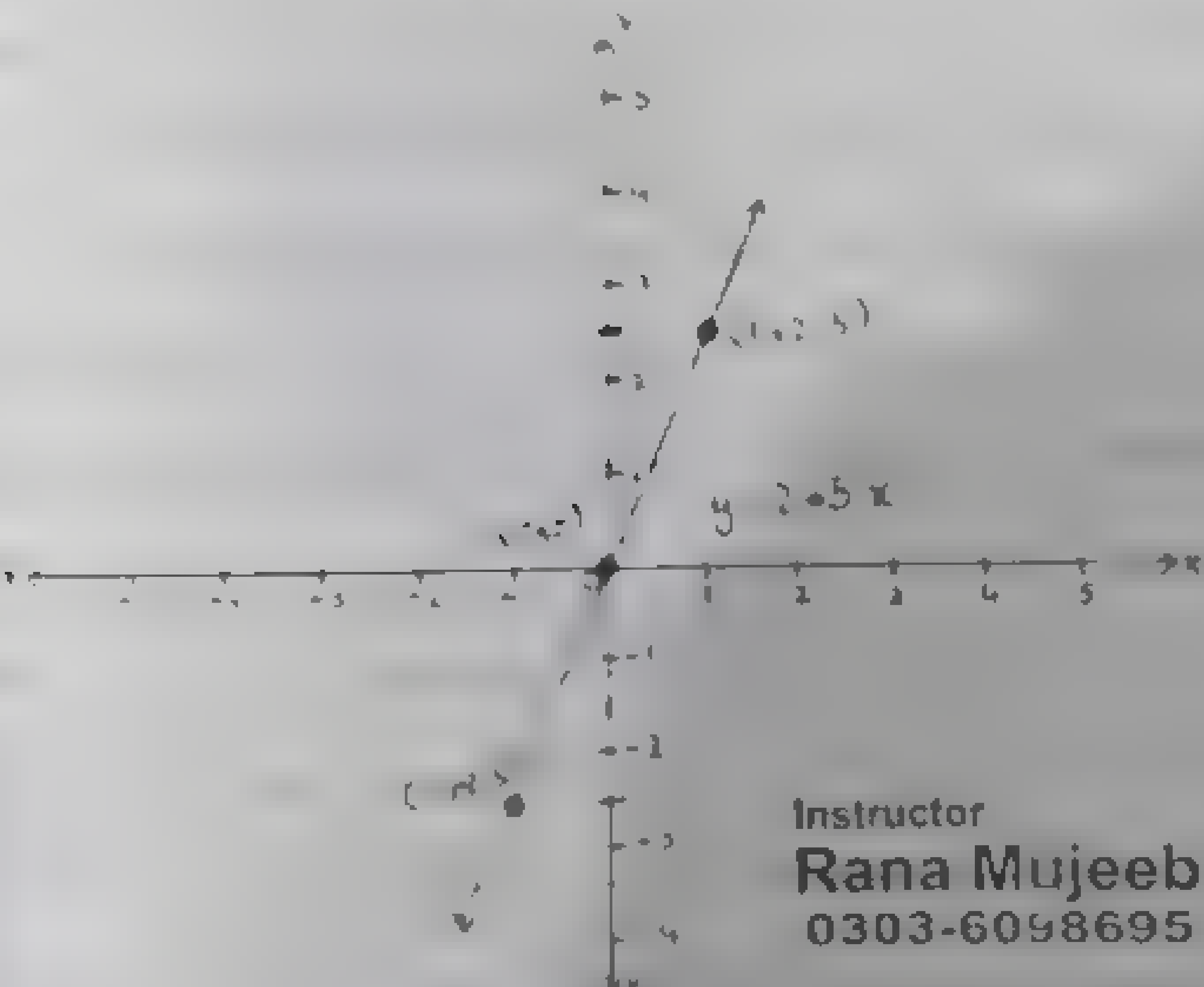
x	1	-1	0
y	[illegible]	[illegible]	0



ii) $y = 5x$

Table:-

	1	
		0



Chapter No 9-

"Introduction to Coordinate Geometry."

Basic Concepts:-

(i) Plane Geometry.
(ii) Coordinate geometry.
(iii) Collinear & non-collinear points.
(iv) Triangle.
(v) Types of triangle.
(vi) Parallelogram.
(vii) Rectangle.
(viii) Square.
(ix) Distance Formula.
(x) Mid-point Formula.
(xi) Ex 9.1.
(xii) Ex 9.2 (Q1, 2, 3, 4, 6, 10 (only))
(xiii) Ex 9.3 (Q1 (only))
(xiv) Review Ex 9.

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(i) Plane Geometry:-

• The study of geometrical shapes in a plane is called plane geometry.

e.g.,

(ii) Coordinate Geometry:-

Coordinate geometry is the study of geometrical shapes in the Cartesian plane (coordinate plane.).

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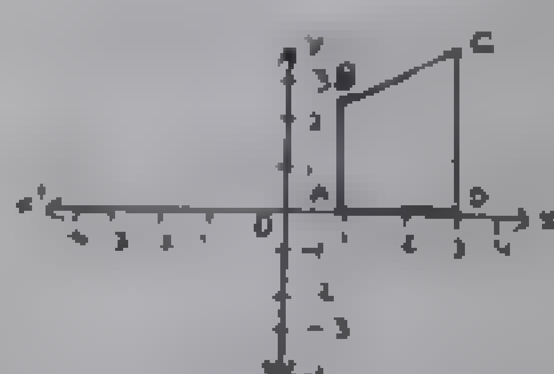
e.g.,

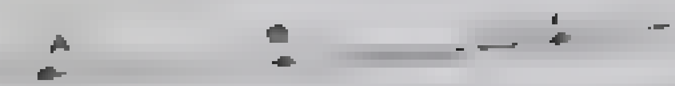


(iii) Collinear & non-collinear points:-

• Collinear points:-

The points which lie on the same straight line is called collinear points.

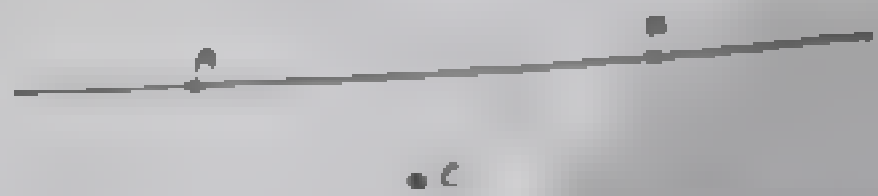
e.g.,



Here, A, B, C are collinear points

- **Non - Collinear Points:-** The points which do not lie on the same straight line is called non collinear points

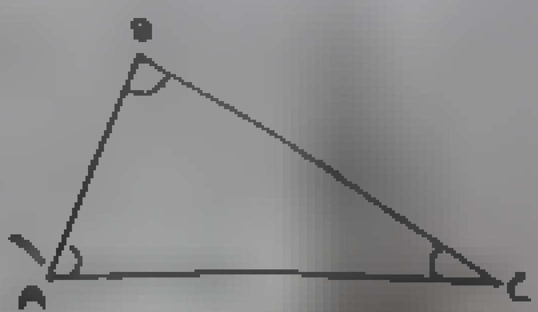
e.g.,



Here, A, B are collinear and C is non collinear.

(iv) Triangle:- A geometrical closed shape having three sides and three angles is called triangle.

e.g.,



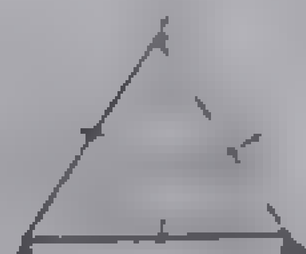
(v) Types of triangles.-

• By sides.-

(a) Equilateral triangle.-

A triangle with all the sides of equal length is called equilateral triangle.

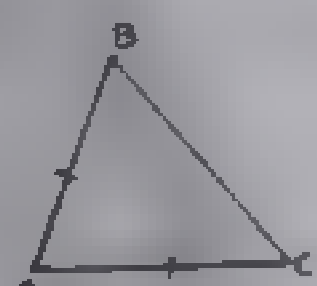
e.g.,



(b) Isoscele triangle.-

A triangle with two sides of equal length is called isoscele triangle.

e.g.,



(c) Scalene triangle:-

A triangle with all the sides of different length is called scalene triangle

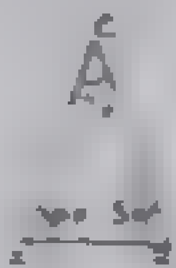
e.g.,

- By Angles.–

(a) Acute angled triangle–

[illegible]

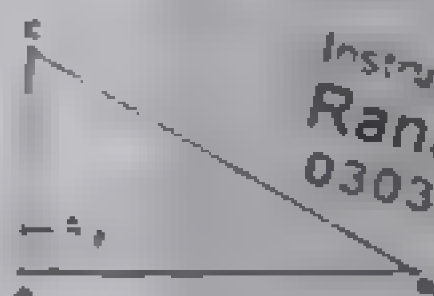
e.g.



(b) Right angled triangle–

A triangle with [illegible]

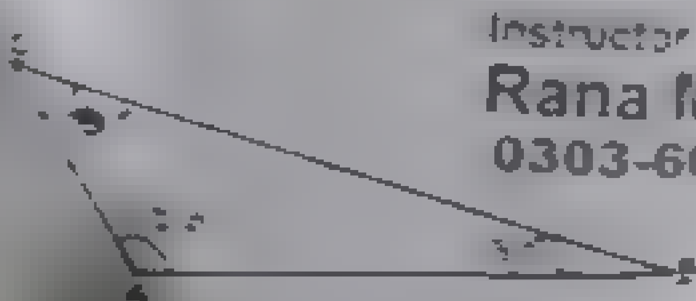
e.g.



(c) Obtuse angled triangle–

A triangle with [illegible]

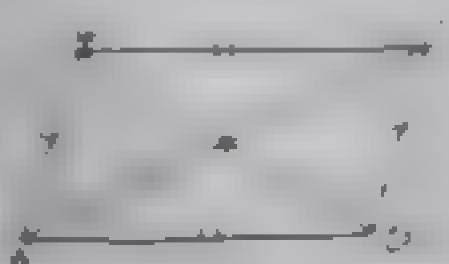
e.g.



(vi) Parallelogram:-

- Opposite sides are parallel.
- Opposite angles are equal.
- The diagonals bisect each other.

e.g.,

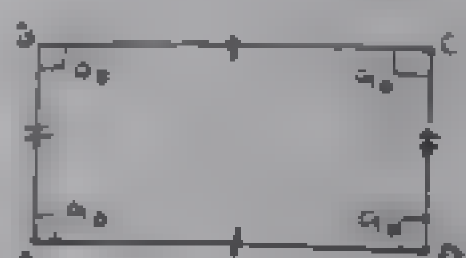


(vii) Rectangle:-

A figure formed in the plane by four non-collinear points is called rectangle if,

- Its opposite sides are equal in length
- The angle at each vertex is of measure 90°.

e.g.,

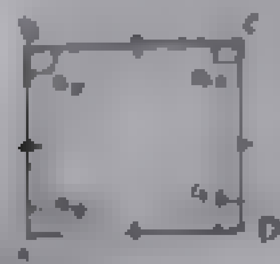


(viii) Square:-

A square is a closed figure in the plane formed by four non-collinear points such that lengths of all sides

are equal and measure of each angle is 90°.

e.g.,



(ix) Distance Formula:-

If $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points and d is the distance between them, then

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(x) Mid-point Formula:-

If $P(x_1, y_1)$ and $Q(x_2, y_2)$ are two points in the plane, then the mid-point $R(x, y)$ of the line segment PQ, is

$$R(x, y) = R\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

(xi) _ Ex 9.1:-

1:- Find the distance between the following pairs of points.

(a) A(9,2), B(7,2)

$|AB| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| \sqrt{(7-9)^2 + (2-2)^2}$

$|AB| = \sqrt{(-2)^2 + (0)^2}$

$|AB| = \sqrt{4+0}$

$|AB| = \sqrt{4}$

$\boxed{|AB| = 2}$

(b) A(2,-6), B(3,-6)

$|AB| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{(3-2)^2 + (-6-(-6))^2}$

$|AB| = \sqrt{(1)^2 + (-\cancel{6}+\cancel{6})^2}$

$|AB| = \sqrt{1 + (0)^2}$

$|AB| - \sqrt{1+0}$

$|AB| = \sqrt{1}$

$\boxed{|AB| = 1}$

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0303-6098695

(c) A (-8,1), B(6,1).

$|AB| - \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{(6-(-8))^2 + (1-1)^2}$

$|AB| = \sqrt{(6+8)^2 + (0)^2}$

[illegible]

$\boxed{|AB| = \text{[illegible]}}$

(d) $A(-4, \sqrt{2})$, $B(-4, -3)$

$|AB| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{(-4-(-4))^2 + (-3-\sqrt{2})^2}$

$|AB| = \sqrt{(-4+4)^2 + (-3-\sqrt{2})^2}$

$|AB| = \sqrt{(0)^2 + (-3-\sqrt{2})^2}$

$|AB| = \sqrt{0 + (-3-\sqrt{2})^2}$

$|AB| = \sqrt{(-3-\sqrt{2})^2}$

$|AB| = \sqrt{(-1)^2(3+\sqrt{2})^2}$

$|AB| = \sqrt{1(3+\sqrt{2})^2}$

$\boxed{|AB| = 3+\sqrt{2}}$

(e) $A(3, -11)$, $B(3, -4)$

$|AB| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{(3-3)^2 + (-4-(-11))^2}$

$|AB| = \sqrt{(0)^2 + (-4+11)^2}$

$|AB| = \sqrt{0 + (7)^2}$

$|AB| = \sqrt{(7)^2}$

$\boxed{|AB| = 7}$

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(f) $A(0,0)$, $B(0,-5)$

$|AB| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{(0)^2 + (-5)^2}$

$|AB| = \sqrt{25}$

$\boxed{|AB| = 5}$

2:- Let P be the point on x-axis with x-component a and Q be the point on y-axis with y-coordiant b as given below. Find the distance b/w P and Q.

(i) $a = 9$, $b = 7$

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0303-6098695

Here

P is $(9,0)$, Q is $(0,7)$.

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$d = \sqrt{(0-9)^2 + (7-0)^2}$

$d = \sqrt{(-9)^2 + (7)^2}$

$d = \sqrt{81 + 49}$

$\boxed{d = \sqrt{130}}$

(ii) $a = 2, b = 3$

Here,

P is $(2,0)$, Q is $(0,3)$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d = \sqrt{(0-2)^2+(3-0)^2}$

$d = \sqrt{(-2)^2+(3)^2}$

$d = \sqrt{4+9}$

$\boxed{d = \sqrt{13}}$

(iii) $a = -8, b = 6$

Here,

P is $(-8,0)$, Q is $(0,6)$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d = \sqrt{(0-(-8))^2+(6-0)^2}$

$d = \sqrt{(0+8)^2+(6)^2}$

$d = \sqrt{(8)^2+36}$

$d = \sqrt{64+36}$

$d = \sqrt{100}$

$\boxed{d = 10}$

(iv) $a = -2, b = -3$

Here,

P is $(-2,0)$, Q is $(0,-3)$

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d = \sqrt{(0-(-2))^2+(-3-0)^2}$

$d = \sqrt{169}$

(v) $a = \sqrt{2}$, $b = 1$

Here,

P is $(\sqrt{2}, 0)$, Q is $(0, 1)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$d = \sqrt{(0 - \sqrt{2})^2 + (1 - 0)^2}$

$d = \sqrt{(-\sqrt{2})^2 + (1)^2}$

$d = \sqrt{2 + 1}$

$\boxed{d = \sqrt{3}}$

(vi) $a = -9$, $b = -4$

Here,

P is $(-9, 0)$, Q is $(0, -4)$

$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$d = \sqrt{(0 - (-9))^2 + (-4 - 0)^2}$

$d = \sqrt{(0 + 9)^2 + (-4)^2}$

$d = \sqrt{(9)^2 + 16}$

$d = \sqrt{81 + 16}$

$\boxed{d = \sqrt{97}}$

(xiii) Ex 9.2 :—

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1:- Let [illegible]

[illegible]

Here, [illegible]

two sides are equal [illegible]

is an isosceles triangle

2:- Let,

A is [illegible], B is [illegible], C is [illegible],

D is [illegible]

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(5 \; [illegible])^2 + (4-1)^2}$ [illegible]

$|BC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|BC| = \sqrt{(2-5)^2 + (-2 \; [illegible])^2}$ [illegible]

$|CD| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|CD| =$ [illegible]

$|AD| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AD| =$ [illegible] $= \sqrt{9+0} = \sqrt{9} = 3$

Here, three sides are equal. So,

it is not a square

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3:- Let

A is $(1,3)$, B is $(4,2)$, [illegible]

[illegible]

$|AB| =$ [illegible]

[illegible]

$|BC| =$ [illegible]

$|CA| =$ [illegible]

$|CA| =$ [illegible]

According to Pythagoras theorem,

$(Hyp)^2 = (Base)^2 + (Perp)^2$

Now,

$|AB|^2 = 10$

$|BC|^2 = 52$

$|CA|^2 = 18$

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Hence,

(i) $10 + 18 \neq 52$

(ii) $10 + 52 \neq 18$

(iii) $18 + 52 \neq 10$

So, it is not a right angled triangle.

4:- Here,

A is $(4,10)$, B is $(1,1)$, C is $(-2,-5)$.

$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|AB| = \sqrt{(1-4)^2 + (1-10)^2} = \sqrt{9 + 81} = \sqrt{90}$

$|BC| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

[illegible] $= \sqrt{9+81} = \sqrt{90}$

$|CA| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|CA| = \sqrt{(4-(-2))^2+(0-(-18))^2} = \sqrt{36+324} = \sqrt{360}$

As,

$$|AB| + |BC| = |CA|$$

$$\sqrt{90} + \sqrt{90} = \sqrt{360}$$

$$\sqrt{9\times10} + \sqrt{9\times10} = \sqrt{36\times10}$$

$$3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10}$$

$$6\sqrt{10} = 6\sqrt{10}$$

So, the points A, B, C are collinear

6:— Here,

C is $(-2, 15)$, A is $(0, 7)$, B is $(3, -5)$

$|CA| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|CA| = \sqrt{(0-(-2))^2+(7-15)^2} = \sqrt{4+64} = \sqrt{68}$

$|AB| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|AB| = \sqrt{(3-0)^2+(-5-7)^2} = \sqrt{9+144} = \sqrt{153}$

$|CB| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|CB| = \sqrt{(3-(-2))^2+(-5-15)^2} = \sqrt{25+400} = \sqrt{425}$

As,

$$|CA| + |AB| = |CB|$$

$$\sqrt{68} + \sqrt{153} = \sqrt{425}$$

$$\sqrt{4\times17} + \sqrt{9\times17} = \sqrt{25\times17}$$

$$2\sqrt{17} + 3\sqrt{17} = 5\sqrt{17}$$

$$5\sqrt{17} = 5\sqrt{17}$$

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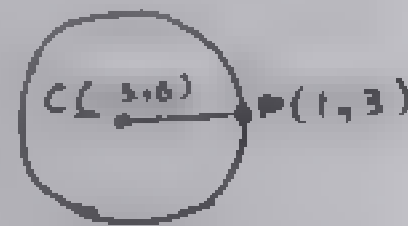
So, the points A, B, C are collinear

10:— Here,

C is $(-3, 6)$, P is $(1, 3)$



$|CP| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|CP| = \sqrt{(1-(-3))^2 + (3-6)^2}$

$|CP| = \sqrt{(1+3)^2 + (-3)^2}$

$|CP| = \sqrt{(4)^2 + 9}$

$|CP| = \sqrt{16+9}$

$|CP| = \sqrt{25}$

$|CP| = 5$

Diametre $= 2 \times |CP|$

Diametre $= 2 \times 5$

$\boxed{\text{Diametre} = 10}$

(viii) Ex 9.3:—

1:— Find the mid-point of the line segment joining each of the following pairs of points:—

(a) $A(9, 2)$, $B(7, 2)$

Mid-point of $\overline{AB} = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$\left(\frac{9+7}{2}, \frac{2+2}{2}\right)$

$\left(\frac{16}{2}, \frac{4}{2}\right)$

Mid point of AB $(8, 2)$

(b) A(2, -6), B(3, -6)

Mid point of AB $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

" " " $\left(\frac{2+3}{2}, \frac{-6+(-6)}{2}\right)$

" " " $= \left(\frac{5}{2}, \frac{-12}{2}\right)$

" " " $= \left(2.5, \frac{-12}{2}\right)$

Mid point of AB $= (2.5, -6)$

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0303-6098695

(c) A(-8, 1), B(6, 1)

Mid-point of $\overline{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

" " " $= \left(\frac{-8+6}{2}, \frac{1+1}{2}\right)$

" " " $= \left(\frac{-2}{2}, \frac{2}{2}\right)$

Mid-point of AB $= (-1, 1)$

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(d) A(-4, 9), B(-4, -3)

Mid-point of $\overline{AB} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

" " $= \left(\frac{-4+(-4)}{2}, \frac{9+(-3)}{2}\right)$

Mid point of $\overline{AB}$ = $(4, 3)$

(e) $A(3, -11)$, $B(3, -4)$

Mid point of $\overline{AB}$ = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

" " " = $\left(\frac{3+3}{2}, \frac{-11+(-4)}{2}\right)$

" " " = $\left(\frac{6}{2}, \frac{-11-4}{2}\right)$

" " " = $\left(3, -\frac{15}{2}\right)$

Mid point of $\overline{AB}$ = $(3, -7.5)$

(f) $A(0, 0)$, $B(0, -5)$

Mid-point of $\overline{AB}$ = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

" " " = $\left(\frac{0+0}{2}, \frac{0+(-5)}{2}\right)$

" " " = $\left(\frac{0}{2}, \frac{0-5}{2}\right)$

" " " = $\left(0, -\frac{5}{2}\right)$

Mid-point of $\overline{AB}$ = $(0, -2.5)$

(xiv) Review Ex 9:-

1:- Choose the correct answers.

(i) Distance b/w points (0,0) and (1,1) is;
(a) 0 (b) 1 (c) 2 (d) $\sqrt{2}$

(ii) Distance b/w points (1,0) and (0,1) is;
(a) 0 (b) 1 (c) $\sqrt{2}$ (d) 2

(iii) Mid-point of points (2,2) and (0,0) is;
(a) (1,1) (b) (1,0) (c) (0,1) (d) (-1,-1)

(iv) Mid point of points (2,-2) and (-2,2) is;
(a) (2,2) (b) (-2,-2) (c) (0,0) (d) (1,1)

(v) A triangle having all sides equal is called;
(a) Isoscele (b) Scalene
(c) Equilateral (d) None of these

(vi) A triangle having all sides different is called;
(a) Isoscele (b) Scalence
(c) Equilateral (d) None of these.

2:- Answer the following, which is true and which is false.

(i) A line has two end points. False.
(ii) A line segment has one end point. False
(iii) A triangle is formed from three collinear points. False.

(iv) [illegible] True.

(v) The mid points of each side of a triangle are collinear. True

(vi) All the points that lie on the x-axis are collinear. True.

(vii) Origin is the only point collinear with the points of both the axes separately. True.

3- Find the distance b/w the following pairs of points.

(i) (6,3), (3,-3)

Let,

A is (6,3), B is (3,-3)

$|AB| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{(3-6)^2 + (-3-3)^2}$

$|AB| = \sqrt{(-3)^2 + (-6)^2}$

$|AB| = \sqrt{9 + 36}$

$\boxed{|AB| = \sqrt{45}}$

(ii) (7,5), (1,-1)

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0303-6098695

Let,

A is (7,5), B is (1,-1)

$|AB| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

[illegible]

[illegible]

$|AB| = \sqrt{36+36}$

$|AB| = \sqrt{72}$

$|AB| = \sqrt{36 \times 2}$

$\boxed{|AB| = 6\sqrt{2}}$

(iii) $(0,0), (-4,-3)$

Let,

A is $(0,0)$, B is $(-4,-3)$

$|AB| = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$

$|AB| = \sqrt{(-4-0)^2 + (-3-0)^2}$

$|AB| = \sqrt{(-4)^2 + (-3)^2}$

$|AB| = \sqrt{16+9}$

$|AB| = \sqrt{25}$

$\boxed{|AB| = 5}$

4:- Find the mid-point b/w following pairs of points.

(i) $(6,6), (4,-2)$

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Let,

A is $(6,6)$, B is $(4,-2)$

Mid-point of $\overline{AB} = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

$$\left(\frac{6+4}{2}, \frac{6+(-2)}{2}\right)$$

" " " $= \left(\frac{10}{2}, \frac{6-2}{2}\right)$

" " " $= \left(5, \frac{4}{2}\right)$

Mid point of AB $(5, 2)$

(ii) $(-5,-7), (-7,-5)$

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Let,

A is $(-5,-7)$, B is $(-7,-5)$

Mid point of AB $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

" " " $= \left(\frac{-5+(-7)}{2}, \frac{-7+(-5)}{2}\right)$

" " " $= \left(\frac{-5-7}{2}, \frac{-7-5}{2}\right)$

" " " $= \left(\frac{-12}{2}, -\frac{12}{2}\right)$

Mid point of $\overline{AB}$ $= (-6, -6)$

(iii) $(8, 0), (0, -12)$

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Let,

A is $(8, 0)$, B is $(0, -12)$

Mid-point of $\overline{AB}$ $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

" " " $= \left(\frac{8+0}{2}, \frac{0+(-12)}{2}\right)$

" " " $= \left(\frac{8}{2}, \frac{0-12}{2}\right)$

" " " $= \left(4, -\frac{12}{2}\right)$

Mid-point of $\overline{AB}$ $= (4, -6)$

Chapter # 10
"Congruent Triangles"

Q 1:- Define congruent triangle?

Ans:-

Two triangles are said to be congruent written symbolically as "≅" if their exists a correspondence between them such that all the corresponding sides and angles are congruent i.e.

$$\text{if} \begin{cases} \overline{AB} \cong \overline{DE} \\ \overline{BC} \cong \overline{EF} \\ \overline{CA} \cong \overline{FD} \end{cases} \text{and} \begin{cases} \angle A \cong \angle D \\ \angle B \cong \angle E \\ \angle C \cong \angle F \end{cases}$$

Q2:-

Define correspondence?

Ans:-

[illegible]

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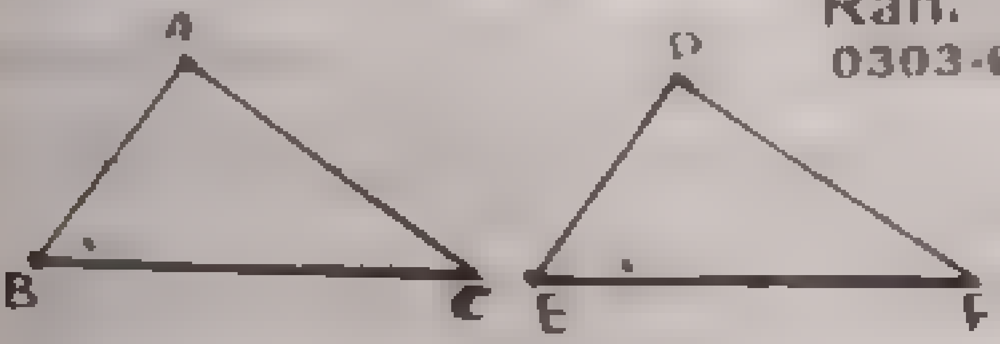

Q3:-

Review Ex 10

1.

Which of the following are true and which are false?

i) A ray has two end points. **False**

ii) In a triangle, there can be only one right angle. **True**

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iii) Three points are said to be collinear if they lie on same line. **True**

iv) Two parallel lines intersect at a point. **False**

v)

Two line can intersect only at one point. **True**

vi)

A triangle of congruent sides has non congruent angles. **False**

2.

If $\Delta ABC \cong \Delta LMN$, then

i)

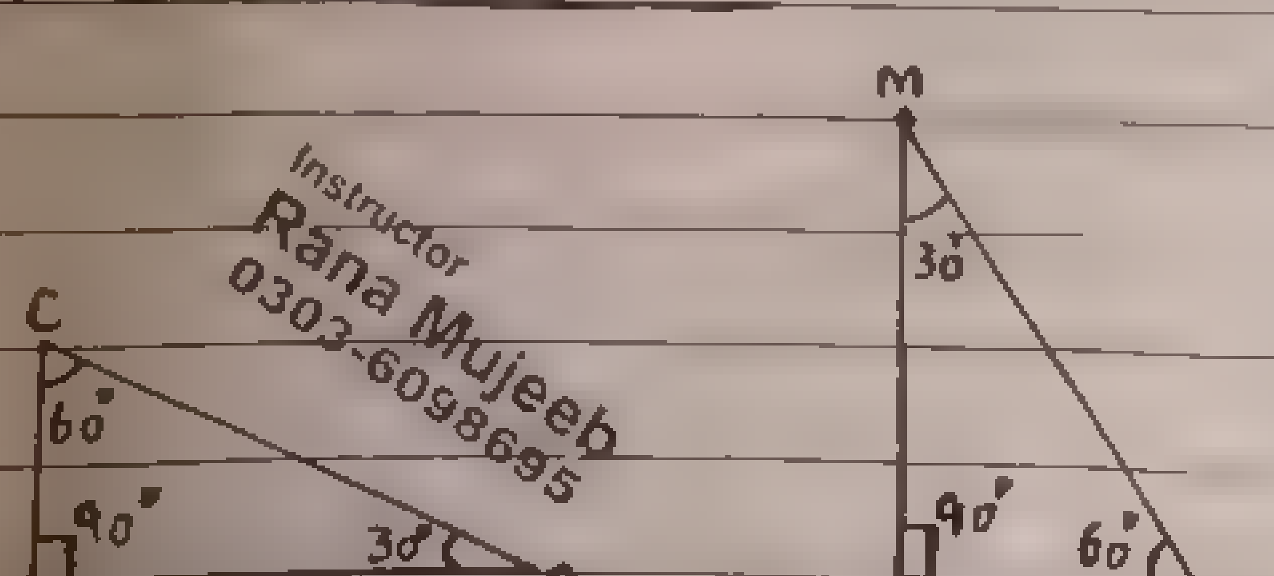
$m\angle M \cong m\angle B$

ii)

$m\angle N \cong m\angle C$

iii)

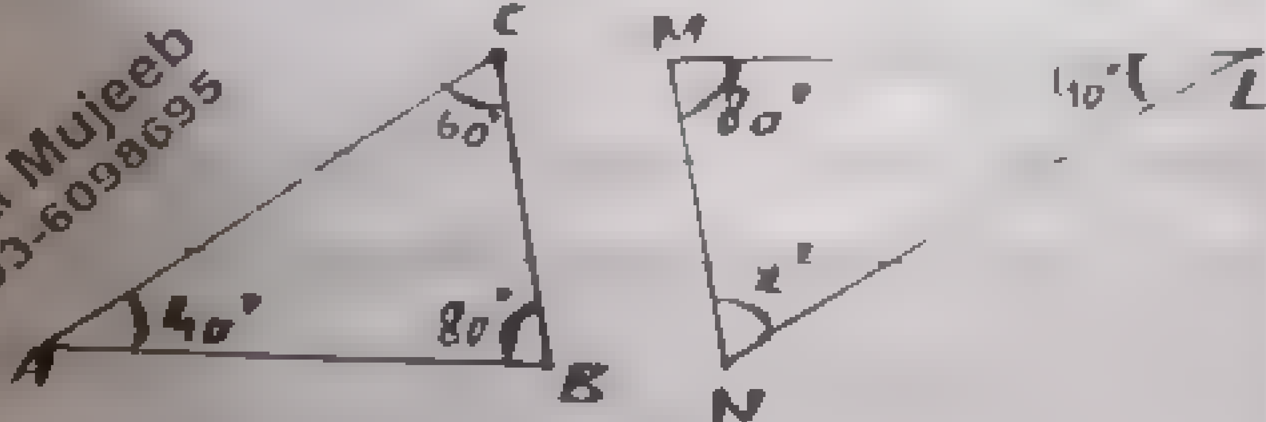
$m\angle A \cong m\angle L$



3.

If $\Delta ABC \cong \Delta LMN$, then find 'x'.

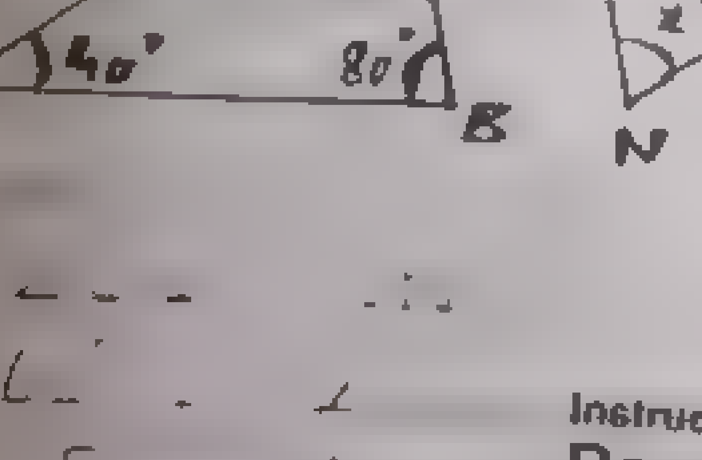
i)



As,

ii)

If ΔLMN, then find 'x'.



$m\angle M + m\angle L + m\angle N = 180°$

$80° + 40° + x = 180°$

$120° + x = 180°$

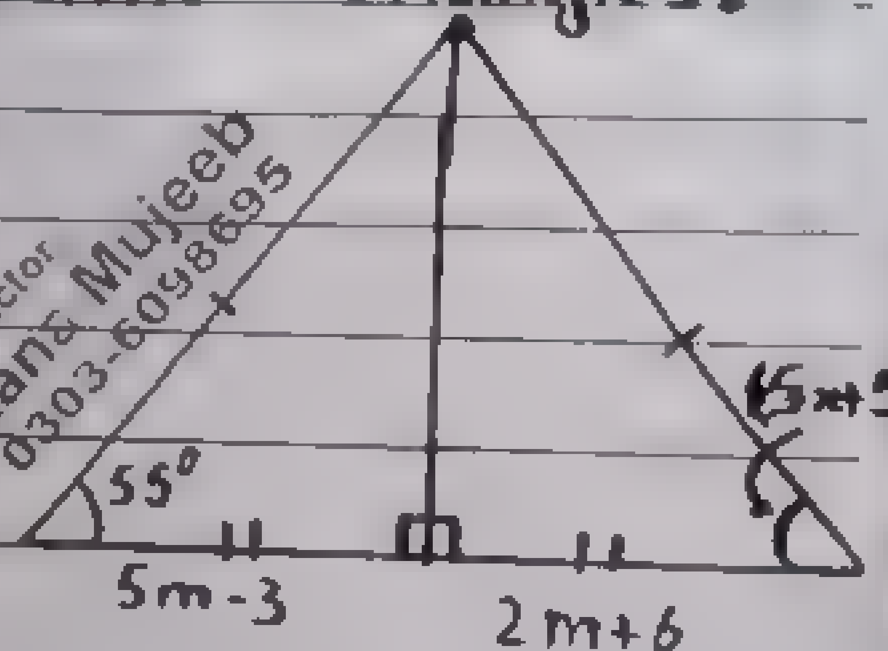
DATE

$x = 180^\circ - 120^\circ$

$\boxed{x = 60^\circ}$

Q.

Find value of unknowns for the given congruent triangles.



As,

$\angle B \cong \angle C$

$55 = 5x + 5$

$55 - 5 = 5x$

$50 = 5x$

$\frac{50}{5} = x$

$10 = x$

$\boxed{x = 10^\circ}$

As,

$\overline{BD} \cong \overline{DC}$

$5m - 3 = 2m + 6$

$5m - 2m = 6 + 3$

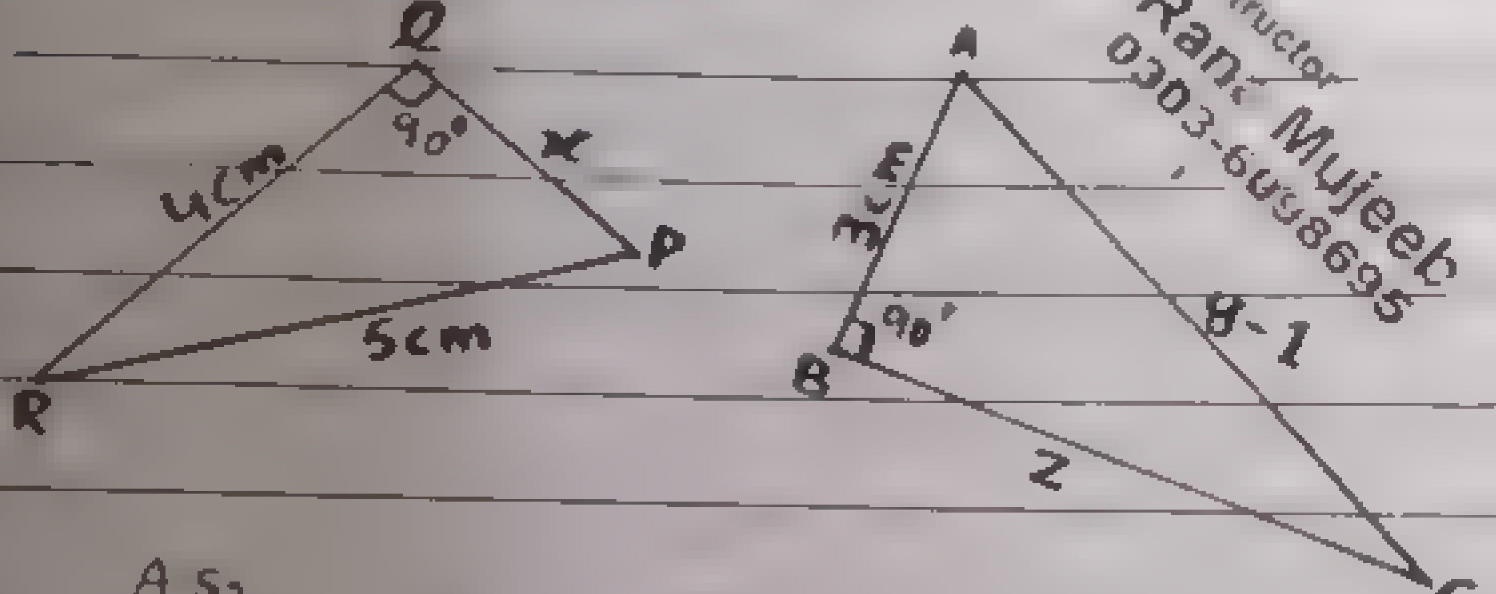
$3m = 9$

$m = \frac{9}{3}$

$\boxed{m = 3}$

5.

If $\Delta PQR \cong \Delta ABC$, then find the unknowns.



As,

$$\overline{PR} \cong \overline{AC}$$

$$5 = y - 1$$

$$5 + 1 = y$$

$$\boxed{y = 6}$$

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As,

$$\overline{PQ} \cong \overline{AB}$$

$$\boxed{x = 3cm}$$

As,

$$\overline{QR} \cong \overline{BC}$$

$$4cm = z$$

$$\boxed{z = 4cm}$$

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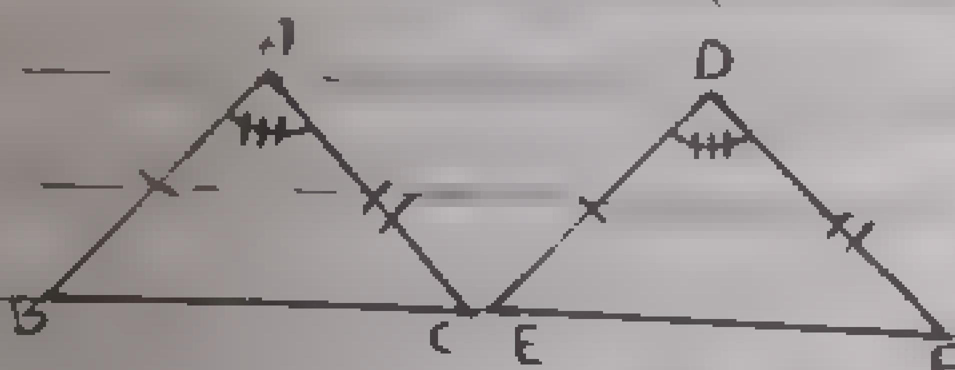
Q4:-

Pastulates.

i)

S.A.S Pastulate:-

[illegible] correspondence of two triangles [illegible] and their included angle of one triangle [illegible] congruent to the [illegible]



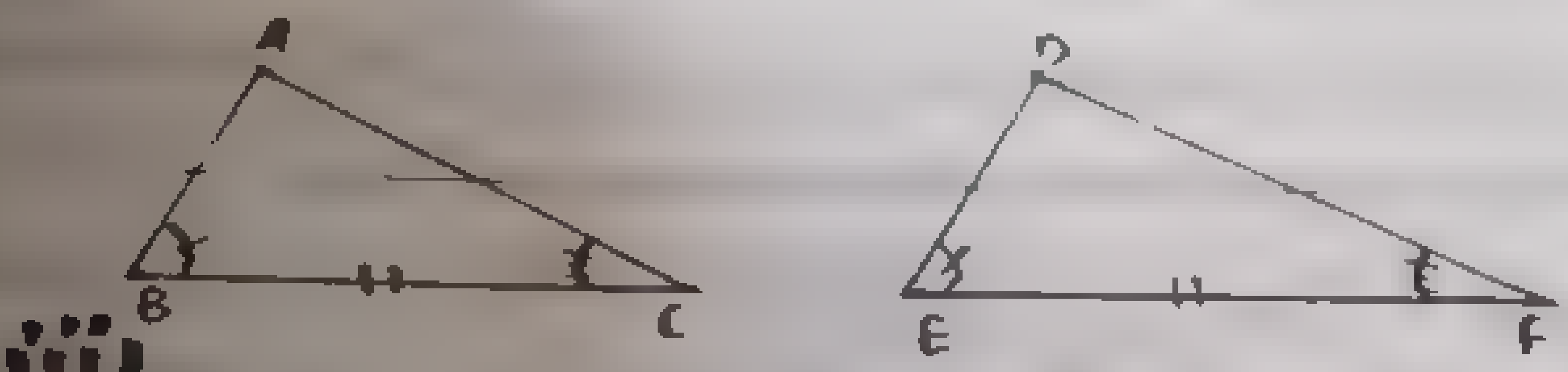
ii)

A.S.A Pastulate:-

If [illegible] [illegible] triangles [illegible] [illegible] two angles [illegible] one [illegible] congruent to [illegible]

corresponding side and angles

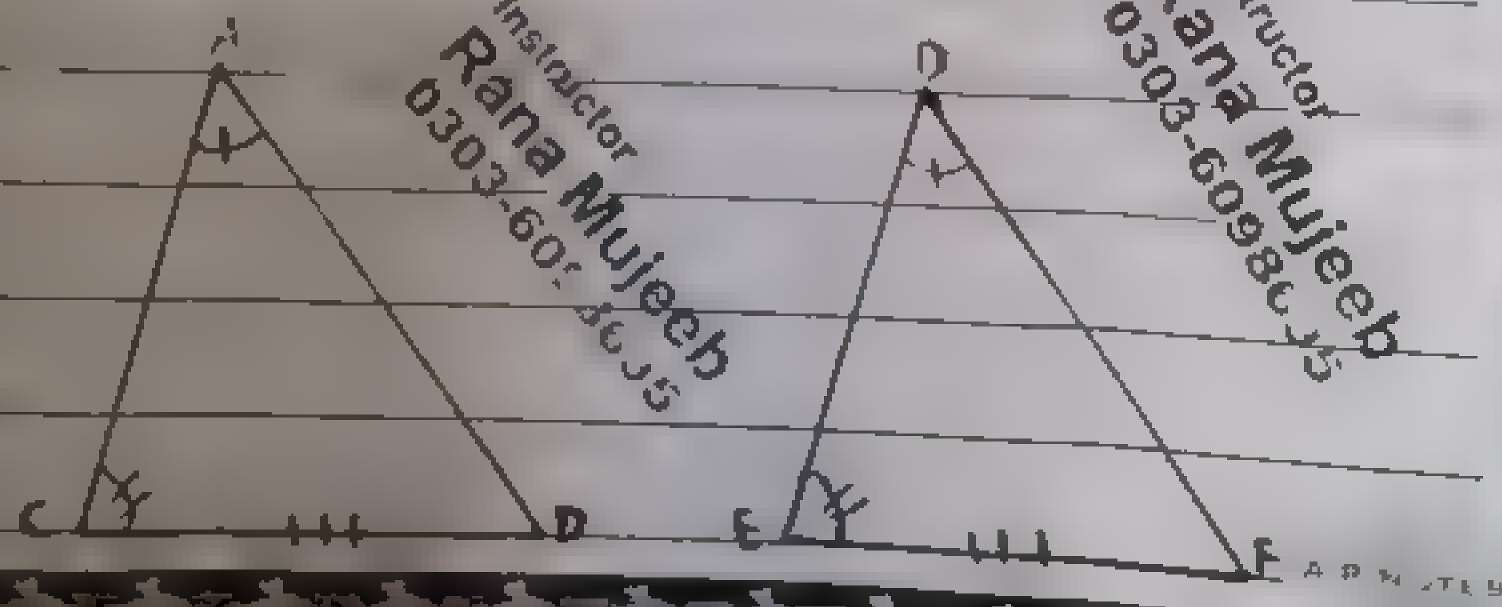
[illegible]



iii)

S.A.S Postulate:-

If [illegible] correspondence [illegible] triangle [illegible] are [illegible] and [illegible] triangle [illegible] are congruent [illegible] corresponding [illegible] other, [illegible] triangles are congruent (S.A.S ≅ S.A.S)



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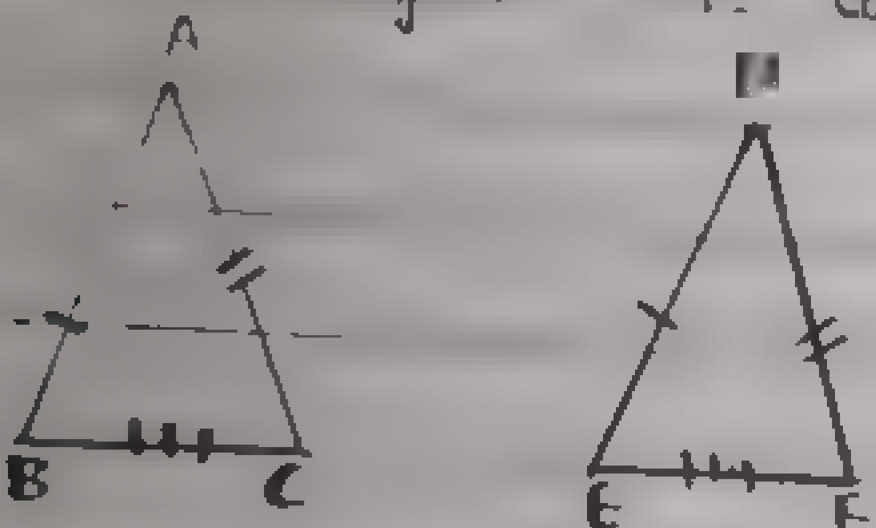
iv)

S.S.S Pastulate.

In a corresponde of two triangles, if three side ... are congruent to the corresponding three sid of ... other, then the two triangles ... congruent ($SSS \cong SSS$).



v)

H.S Pastulate:-

If ... the ... [illegible] ... right ... the ... and ... are ... [illegible]

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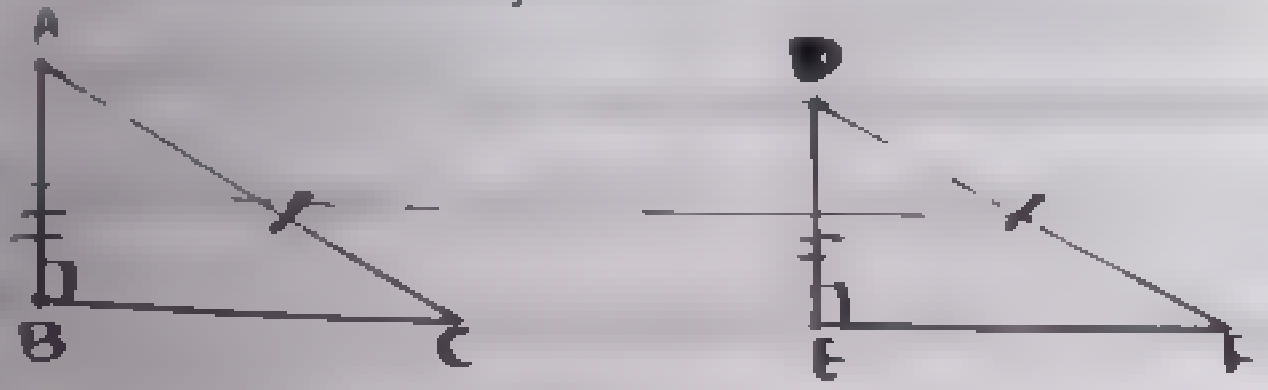
... corresponding side of the othe. then the triangles are congruent

(H.S ≅ H.S)



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Q5:- VIP MCQ's:-

If two angles of a triangle are congruent then the sides opposite to them are also **congruent**.

Q6:- If one angle of right triangle is of 30° than what is its hypotenuse?

Ans:- If one angle of a right triangle is of 30°, the hypotenuse is twice as long as the side opposite to the angle.

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Chapter No 11

"Parallelograms and Triangles"

Basic Concepts

i. Polygons.
ii. Quadrilaterals.
iii. Trapeziums.
iv. Parallelograms.
v. Rhombus.
vi. Rectangles.
vii. Squares.
viii. Review Ex. 11.
ix. Some Important Concepts.

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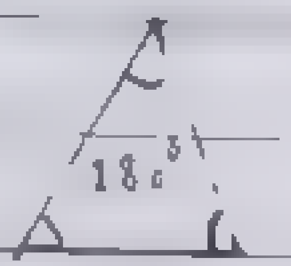
i. Polygons:-

Def:-

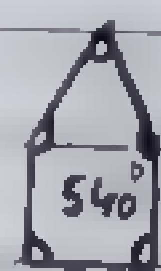
The word "polygon" consist of two words, one is "poly" which means "many" and other is "gon" which means "angles." Polygon is a closed figure formed by line-segments.

Example:-



Triangle

Rectangle



Pentagon

Hexagon

Formula:-

Sum of angles of polygon $= (n-2) \times 180^\circ$

Here, "n" is a number of sides.

ii. Quadrilateral:

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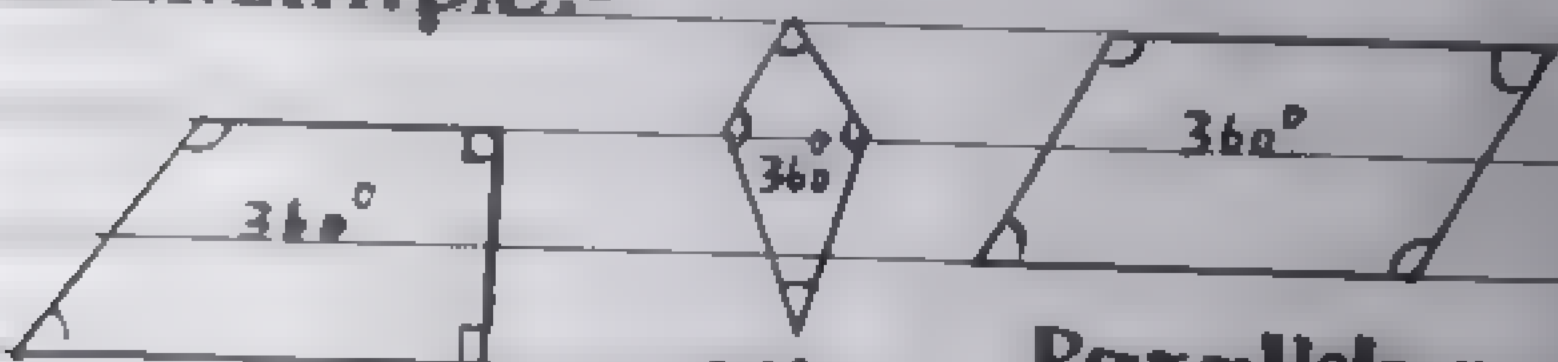
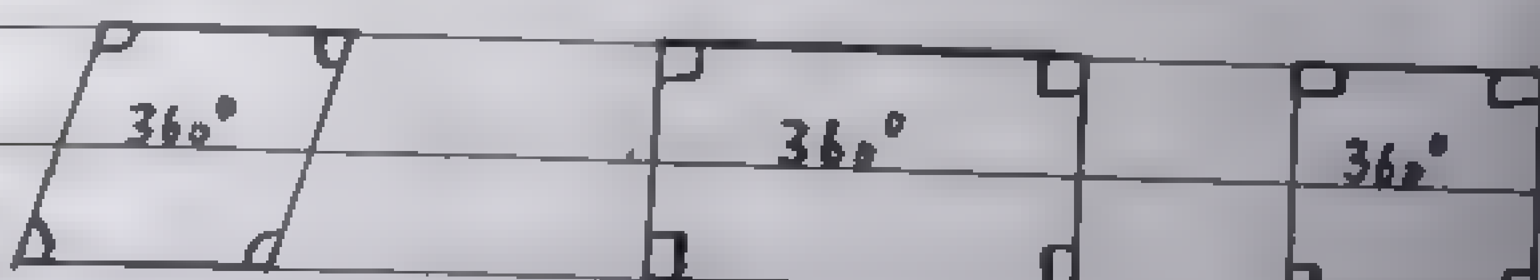
Def:

The word "quadrilateral" consist of two words, one is "quadri" which means "four" and other is "lateral" which means "sides" A polygon with four sides, four angles and four vertices is called quadrilateral

Example:-



Trapezium   Kite   Parallelogram



Rhombus   Rectangle   Square

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iii. Trapezium:

Def:-

i) A quadrilateral with one pair of parallel sides is called trapezium.

OR

ii) A quadrilateral with two parallel sides and two unparallell sides is called trapezium.

Example:-

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Trapeziums.

iv. Parallelgram:-

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Def:-

A figure formed by four non-collinear points in the plane is called a parallelogram If:-

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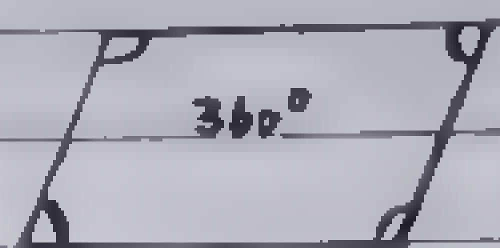
i) Opposite sides are parallel and congruent.

ii) Opposite angles are congruent

iii) The diagonals bisect each other.

Example:-



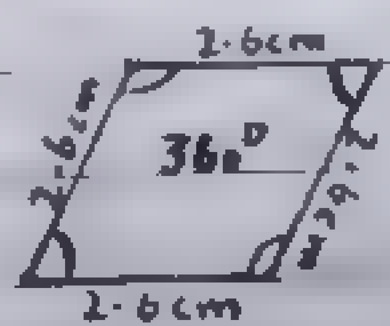
Parallelogram.

v. Rhombus:-

Def:-

A parallelogram with all equal sides is called rhombus.

Example:



Rhombus.

Date:

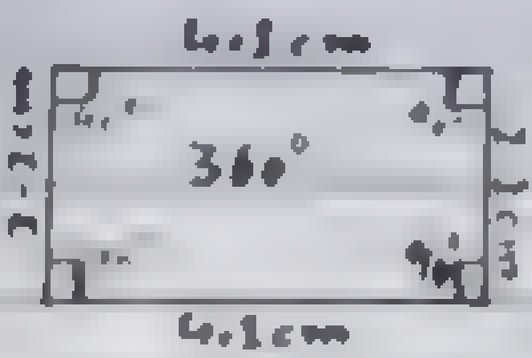
vi. Rectangle:

Def:

A figure formed in the plane by four non-colinear points is called rectangle if:-

i) Its opposite sides are of equal measure
ii) Its opposite sides are parallel to each other
iii) The angle at each vertex is of measure of 90°

Example:



Rectangle.

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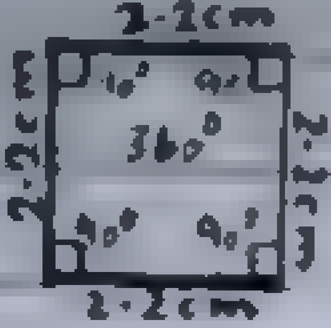
vii. Square:

Def:-

A square is a closed figure in the plane formed by four non-colinear points such that lengths of all sides are equal and measure of each angle is 90°.

Example:-



Square.

viii. Review Ex. 11:-

1. Fill in the blanks.

i) In a parallelogram opposite sides are congruent.

ii) In a parallelogram opposite angles are congruent.

iii) Diagonals of a parallelogram intersect each other at a point.

iv) Medians of a triangle are concurrent.

v) Diagonals of a parallelograme divides the parallelogram into two congruent triangles.

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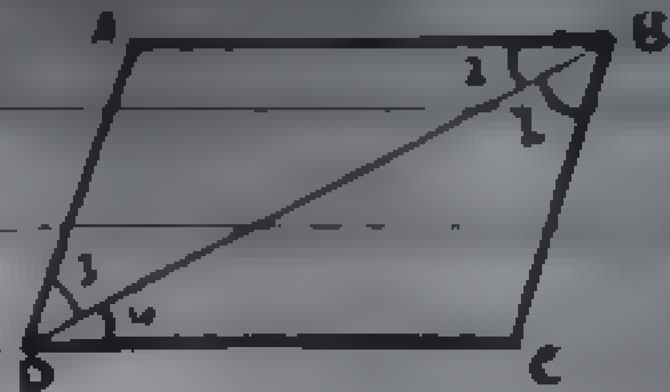
2. In parallelogram ABCD:-



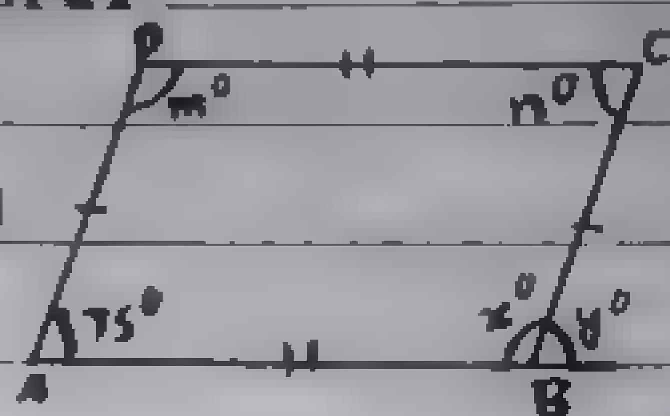
i) $m\overline{AB} \cong m\overline{DC}$.

ii) $m\overline{BC} \cong m\overline{AD}$.

iii) $m\angle 1 \cong m\angle 3$.

iv) $m\angle 2 \cong m\angle 4$.

3. Find the unknowns in the given figure.



As,

$m\angle A = m\angle C$ (opposite angles)

$75^\circ = n$

$\boxed{n = 75^\circ}$

Now,

$y^\circ = n^\circ$ (Alternate angles).

$\boxed{y^\circ = 75^\circ}$

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As,

$m\angle A + m\angle B = 180^\circ$ (suplimentry angles)

$75^\circ + x = 180^\circ$

$x = 180^\circ - 75^\circ$

$\boxed{x = 105^\circ}$

Now,

$m^\circ = x^\circ$ (opposite angles).

$\boxed{m^\circ = 105^\circ}$

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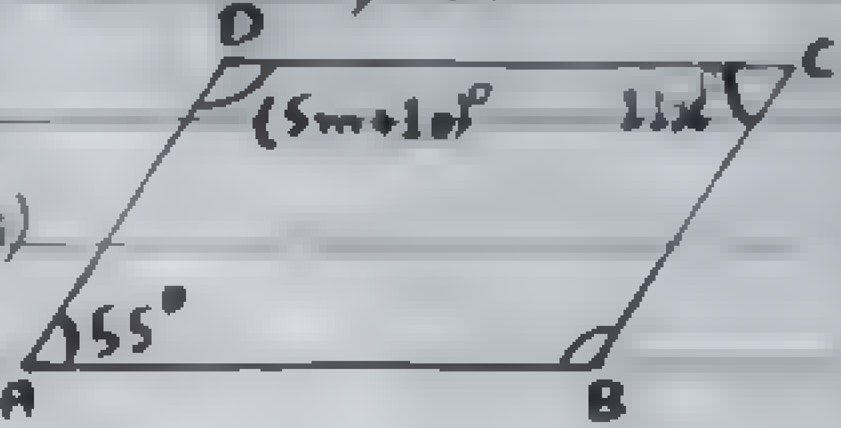
4. If the given figure ABCD is a parallelogram, then find "x, m":-



Ans

$m\angle A = m\angle C$ (opposite angles)

$55^\circ = 11x^\circ$

$\frac{\cancel{55}^{5}{}^\circ}{\cancel{11}_{1}{}^\circ} = x^\circ$

$\boxed{x^\circ = 5^\circ}$

Now,

$m\angle A + m\angle D = 180^\circ$ (suplementry angles)

$55^\circ + (5m + 10)^\circ = 180^\circ$

$55^\circ + (5m)^\circ + 10^\circ = 180^\circ$

$65^\circ + (5m)^\circ = 180^\circ$

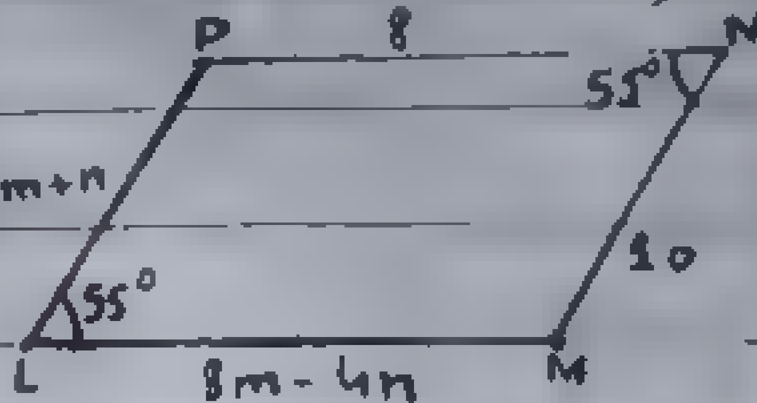
$(5m)^\circ = 180^\circ - 65^\circ$

$(5m)^\circ = 115^\circ$

$m^\circ = \frac{\cancel{115}^{23}{}^\circ}{\cancel{5}_{1}{}^\circ}$

$\boxed{m^\circ = 23^\circ}$

5. The given figure LMNP is a parallelogram. Find value of "m, n".



$m\overline{LP} = m\overline{MN}$ (opposite sides)

$4m+n = 10 \longrightarrow$ (i)

$m\overline{LM} = m\overline{PN}$ (opposite sides)

$8m-4n = 8 \longrightarrow$ (ii)

By multiplying eq (i) by 4,

$4\times(4m+n) = 10\times 4$

$16m+4n = 40 \longrightarrow$ (iii)

By adding eq (ii) and (iii),

$8m - \cancel{4n} = 8$

$16m + \cancel{4n} = 40$

$24m = 48$

$m = \frac{48}{24}$

$\boxed{m=2}$

By putting "m=2" in eq (i),

$4m+n = 10$

$4(2)+n = 10$

$8+n = 10$

$n = 10-8$

$\boxed{n=2}$

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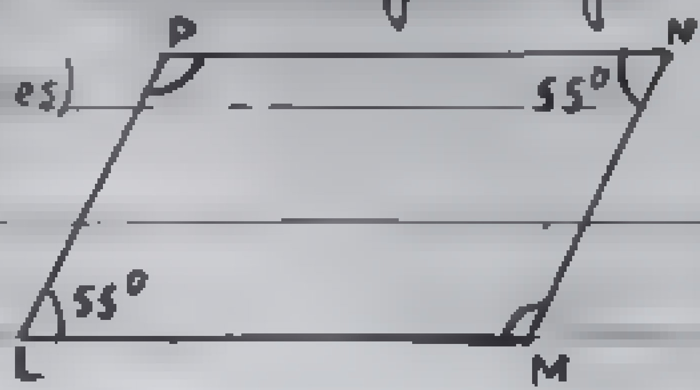
6. In the question 5, sum of the opposite angles of the parallelogram is 110°, find the remaining angles.



$m\angle L + m\angle M = 180^\circ$ (supplementry anges)

$55^\circ + m\angle M = 180^\circ$

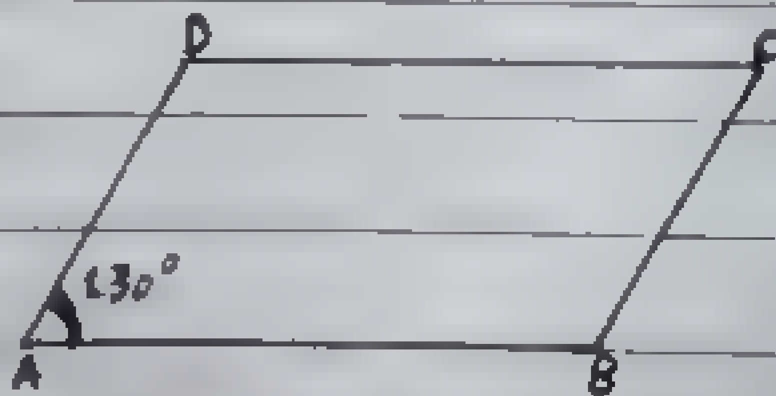
$m\angle M = 180^\circ - 55^\circ$

$\boxed{m\angle M = 125^\circ}$

$m\angle P = m\angle M$ (opposite angles).

$\boxed{m\angle P = 125^\circ}$

7. One angle of parallelogram is 130°, find the measure of remaining angles.



$m\angle C = m\angle A$ (opposite angles)

$\boxed{m\angle C = 130^\circ}$

$m\angle A + m\angle B = 180^\circ$ (Supplementry angles)

$130^\circ + m\angle B = 180^\circ$

$m\angle B = 180^\circ - 130^\circ$

$\boxed{m\angle B = 50^\circ}$

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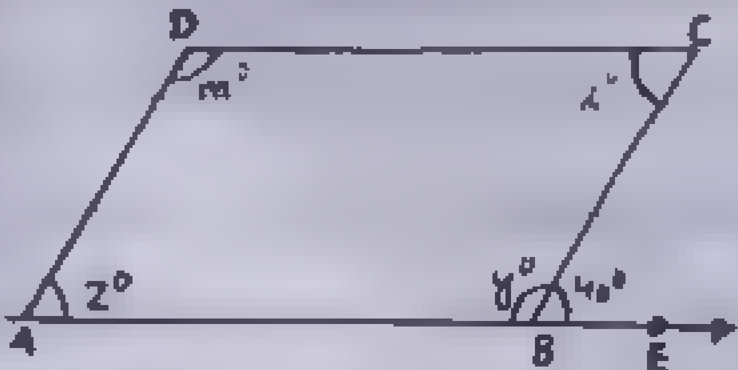
$m\angle D = m\angle B$ (opposite angles)

$\boxed{m\angle D = 50^\circ}$

8. One exterior angle on the one side of parallelogram is $40°$, Find the remaining angles.



$\boxed{x° = 40°}$ (Alternate angles)

$z° = x°$ (opposite angles)

$\boxed{z° = 40°}$

$z° + y° = 180°$ (Supplementry angles)

$40° + y° = 180°$

$y° = 180° - 40°$

$\boxed{y° = 140°}$

$m° = y°$ (opposite angles)

$\boxed{m° = 140°}$

ix. Some Important Concepts:-

1. Diagonals:-

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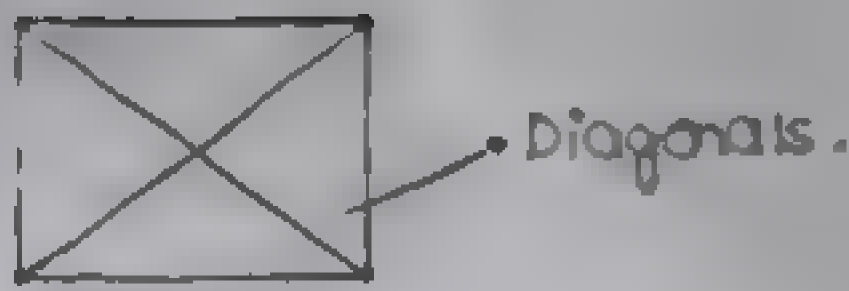
Def:-

A line segment that join two non-adjacent vertices of a polygon is called a diagonals

Examples



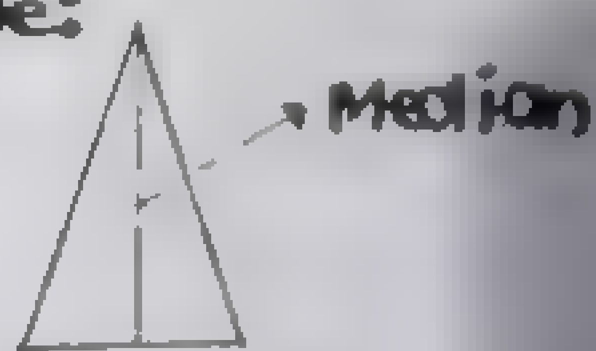
Rectangles.

2. Medians:

Def:-

Median of a triangle is a line segment that join the vertex of triangle to the midpoint of opposite side.

Example:



Triangle

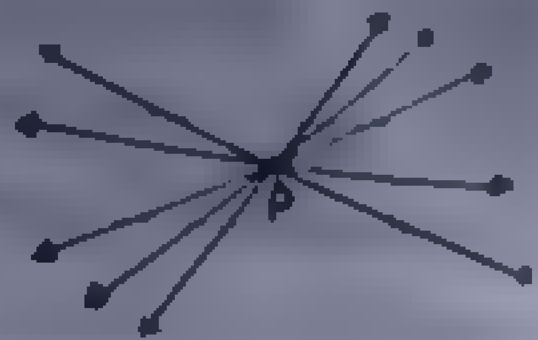
3. Concurrent:

Def:

If a set of lines intersect each other at same

point the line are called concurrent.

Example:-



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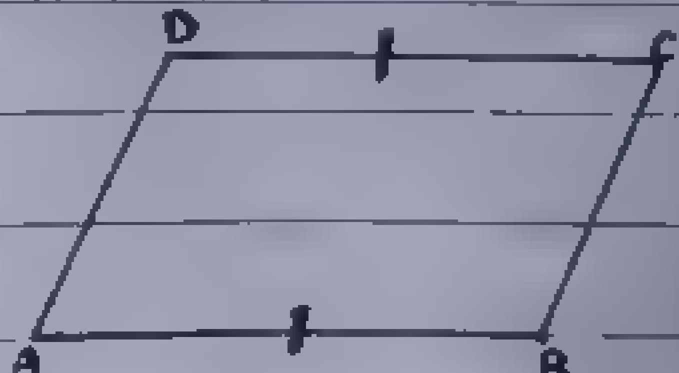
Here, "p" is the point of concurency

4. Theorem 11.1.2:-

If two opposite sides of a quadrilateral are congurent and parallel, it is a parallelogram.

Example:-



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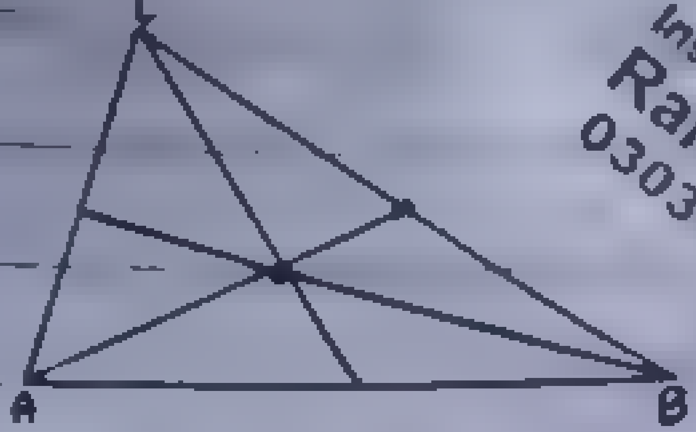
Parallelogram

5. Theorem 11.1.4:-

The medium of a triangle are concurrent and their point of concurrency

is the point of trisection of each medium.

Example:-

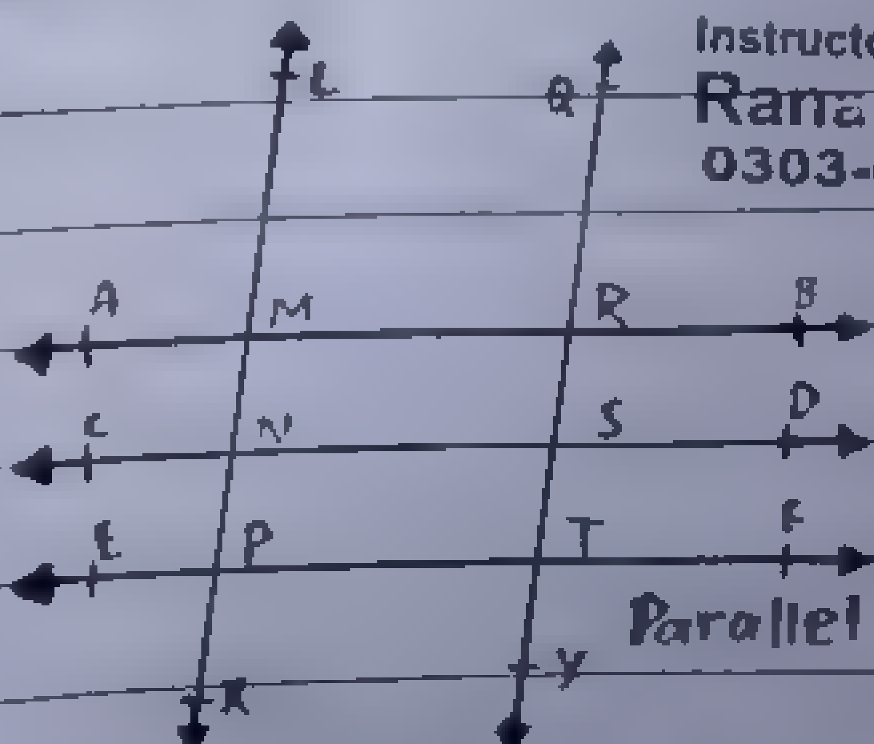


Triangle

6. Theoram 11·1·5:-

If three or more parallel lines make congruent segments on a transversal, they also intercept congruent segments on any other line that cuts them.

Example:-



Parallel lines

Chapter #12:-

"Line Bisectors and Angle Bisectors."

Basic Cocepts:-

(i) Bisector of Line segment.

(ii) Right bisector of line segment.

(iii) Bisector of an angle.

(iv) Theorem 12.1.1.

(v) Theorem 12.1.2.

(vi) Theorem 12.1.3.

(vii) Theorem 12.1.4.

(viii) Theorem 12.1.5.

(ix) Theorem 12.1.6.

(x) Observe that.

(xi) Note.

(xii) Theorem.

(xiii) Parts.

(xiv) Statement.

(xv) Given.

(xvi) To Prove.

(xvii) Figure.

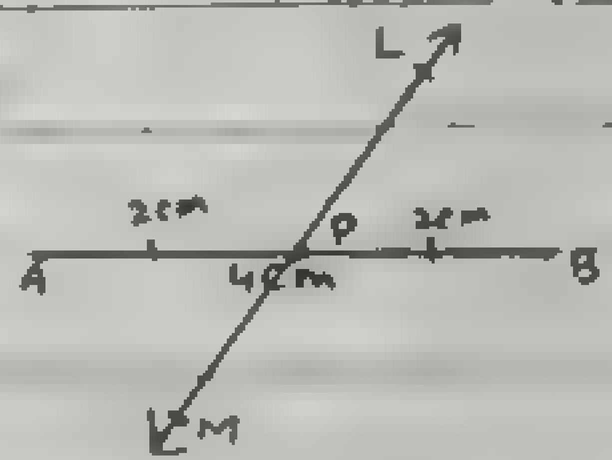
(xviii) Construction.

(xix) Proof.

(xx) Review Exercise 12 (Q1, 2, 4, 5, 6)

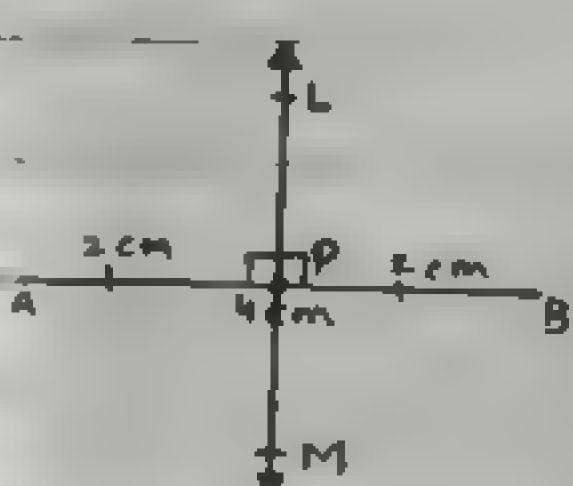
(i) Bisector of Line segment:-

A line l is called a bisector of a line segment if l passes through its mid-point.



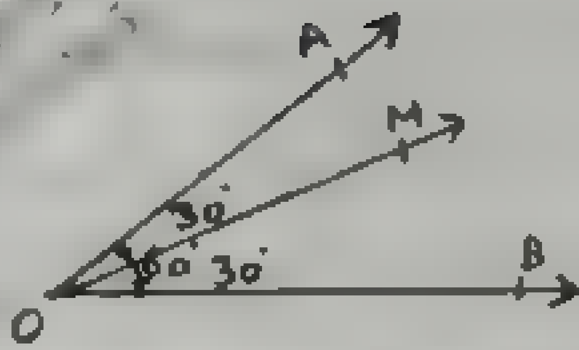
(ii) Right bisector of line segment:-

A line l is called right bisector of line segment if l is perpendicular to the line segment and passes through its mid-point.



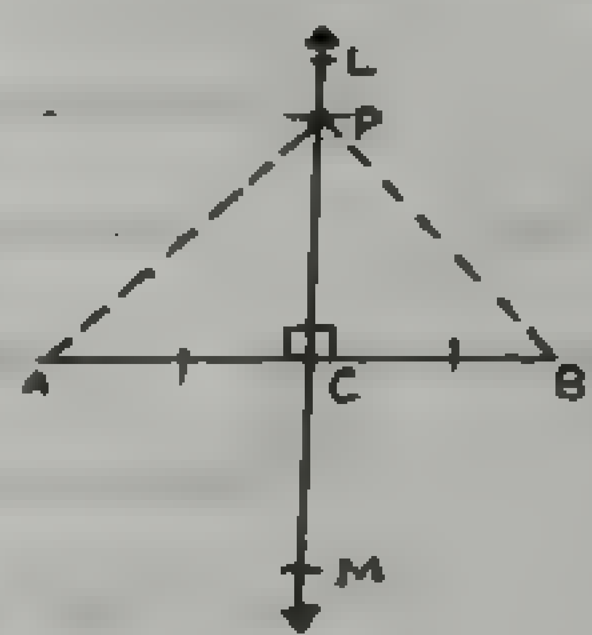
(iii) Bisector of an angle:-

Bisector of an angle is a line or a ray that divides the given angle into two equal parts.



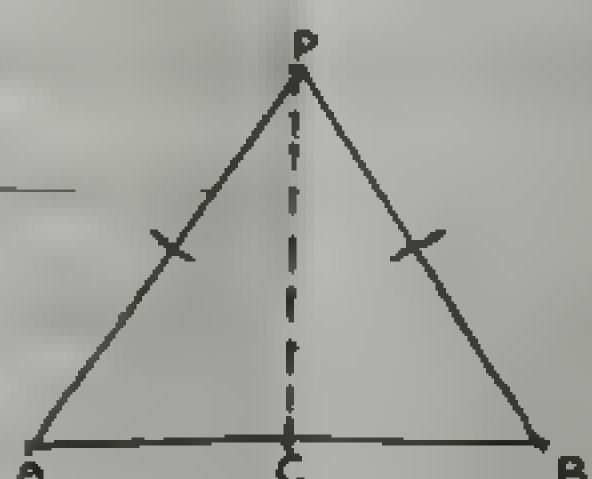
(iv) Theorem 12.1.1:-

Any point on the right bisector of a line segment is equidistant from its end points.



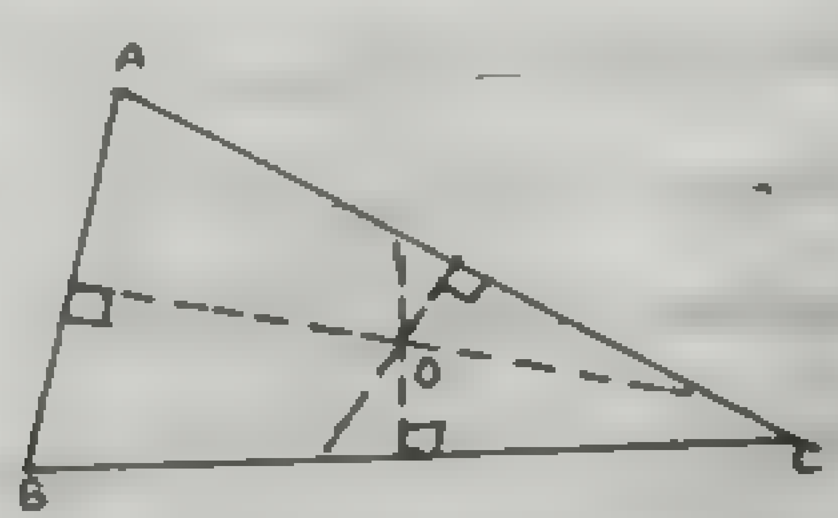
(v) Theorem 12.1.2:-

Any point equidistant from the end points of a line segment is on the right bisector of it.



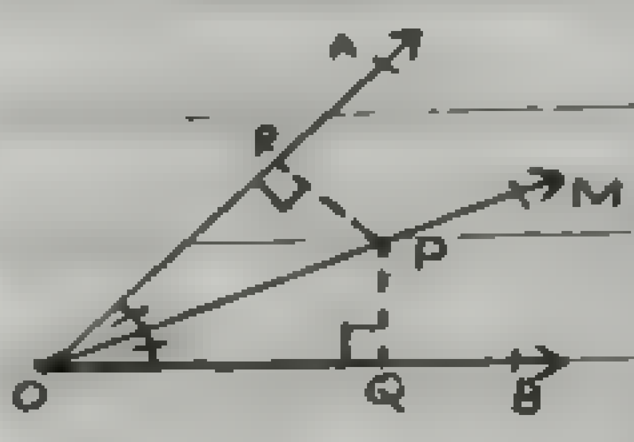
(vi) Theorem 12.1.3:-

The right bisectors of the sides of a triangle are concurrent.



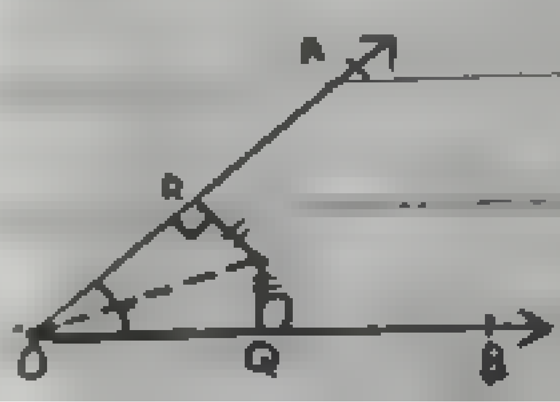
(vii) Theorem 12.1.4:-

Any point on the bisector of an angle is equidistant from its arms.



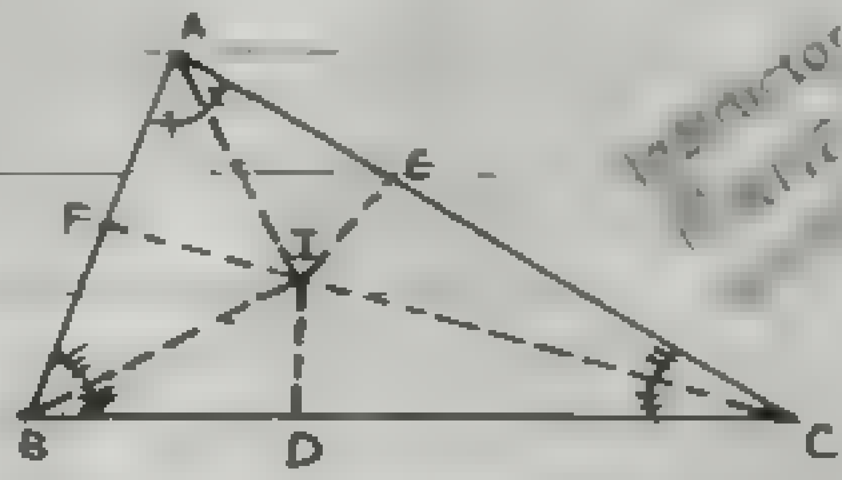
(viii) Theorem 12.1.5:-

Any point inside an angle, equidistant from its arms, is on the bisector of it.



(ix) Theorem 12·1·6:–

The bisectors of the angles of a triangle are concurrent.



(x) Observe that:–

- The right bisectors of the sides of an acute triangle intersect each other inside the triangle
- The right bisectors of the sides of a right triangle intersect each other on the hypotenuse.
- The right bisectors of the sides of an obtuse triangle intersect each othe outside the triangle.

(xi) Note:–

In practical geometry also, by constructing angle bisectors of a triangle, we shall verify that they are concurrent.

(xii) Theorem:–

Theorem is a true statement which can be proven.

(xiii) Parts of a Theorem:–

(a) Statement (b) Given

(c) To Prove (d) Figure

(e) Construction (f) Proof.

(xiv) Statement:–

The description of a theorem in words is called statement.

(xv) Given:–

The condition described in the statement of theorem according to given figure is called given.

(xvi) To Prove:–

The required result of the theorem which is to be proven is called to prove.

(xvii) Figure:-

A complete drawing of theorem according given statement is called figure.

(xviii) Construction:-

The additional work done on the figure in order to prove theorem is called construction.

(xix) Proof:-

The most important part of a theorem which uses statements and reasons in order to prove theorem is called proof.

(xx) Review Exercise 12:-

1. Which of the following are true and which are false?

(i) Bisection means to divide into two equal parts. True.

(ii) Right bisection of line segment means to draw perpendicular which passes through the mid-point of line segment.

True.

(iii) An point on the right bisector of a line segment is not equidistant from its end points. False.

(iv) Any point equidistant from the end points of a line segment is on the right bisector of it. True.

(v) The right bisectors of the sides of a triangle are not concurrent. False.

(vi) The bisectors of the angles of a triangle are concurrent. True.

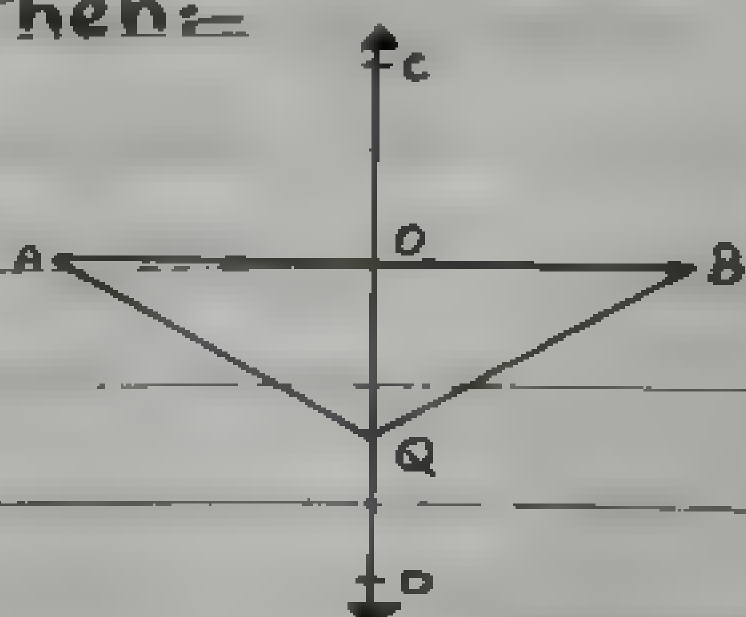
(vii) Any point on the bisector of an angle is not equidistant from its arms. False.

(viii) Any point inside an angle, equidistant from its arms, is on the bisector of it. True.

2. If $\overleftrightarrow{CD}$ is right bisector of line segment $\overline{AB}$, then:—

(i) $m\overline{OA} = m\overline{OB}$.

(ii) $m\overline{AQ} = m\overline{BQ}$.



4. The given triangle ABC is equilateral triangle and $\overline{AD}$ is bisector of angle A, then find the values

of unknowns x°, y° and z°.

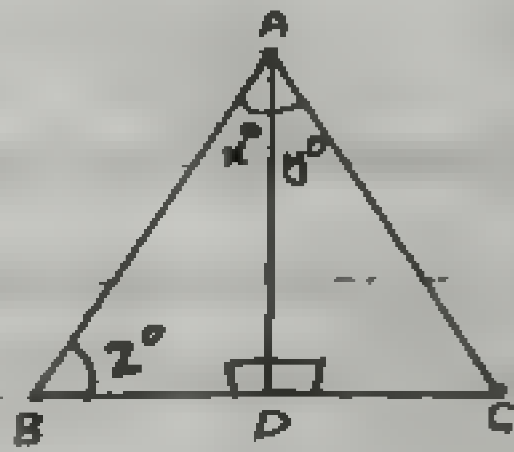
As,

equilateral triangle is also an equiangular triangle.

So,

$m\angle A = m\angle B = m\angle C$

$\boxed{z^\circ = 60^\circ}$



As,

$\overline{AD}$ is a bisector of $\angle A$.

So, $x^\circ = y^\circ = \frac{\cancel{60}^{30}}{\cancel{2}_1} = 30^\circ$

5. In the given congruent triangles LMO and LNO, find the unknowns x and m.

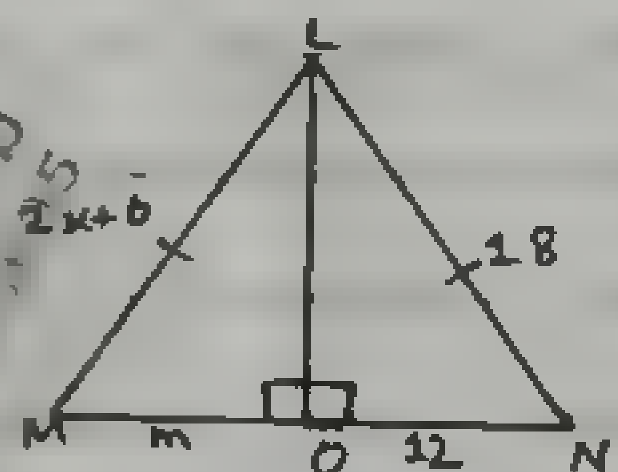
As,

$\triangle LMO \cong \triangle LNO$

So,

$m\overline{OM} = m\overline{ON}$

$\boxed{m = 12}$



Now,

$m\overline{LM} = m\overline{LN}$

$2x + 6 = 18$

$2x = 18 - 6$

$2x = 12$

$x = \frac{\cancel{12}^6}{\cancel{2}_1}$

$\boxed{x = 6}$

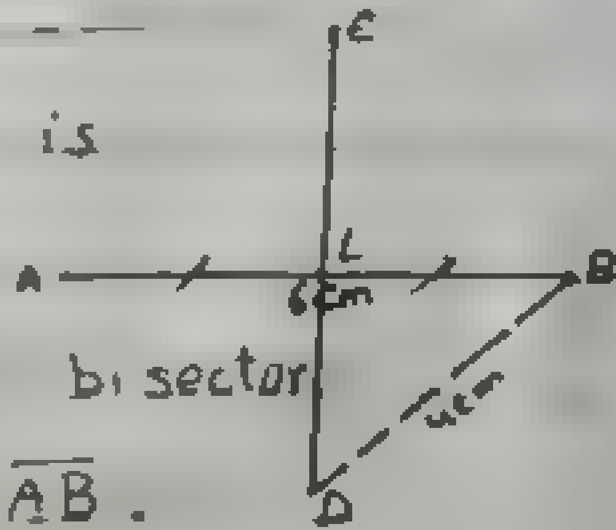
6. $\overline{CD}$ is right bisector of the line segment AB.

(i) If $m\overline{AB} = 6cm$, then find the $m\overline{AL}$ and $m\overline{LB}$.

(ii) If $m\overline{BD} = 4cm$, then find $m\overline{AD}$.

(i) As,

$m\overline{AL} = m\overline{LB} = \frac{6}{2} = 3cm$ ∵ $\overline{CD}$ is the right bisector of $\overline{AB}$.



(ii) As,

$m\overline{AD} = m\overline{BD} = 4cm$ ∵ Any point on the bisector of line segment is equidistant from its end points.

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Chapter # 13

"Sides and Angles of a Triagle"

Basic Cocepts:-

i) Types of Triangles.
ii) Theorem 13.1.1.
iii) Theorem 13.1.2.
iv) Theorem 13.1.3.
v) Theorem.
vi) Theorem 13.1.4.
vii) Corollaries.
viii) Note.
ix) Review Ex. 13.
x) Some Important Cocepts.

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i) Types of Triangles:-

• By Sides:-

a) Equilateral Triangle:-

A triangle with all the sides of equal length is called equilateral triangle.

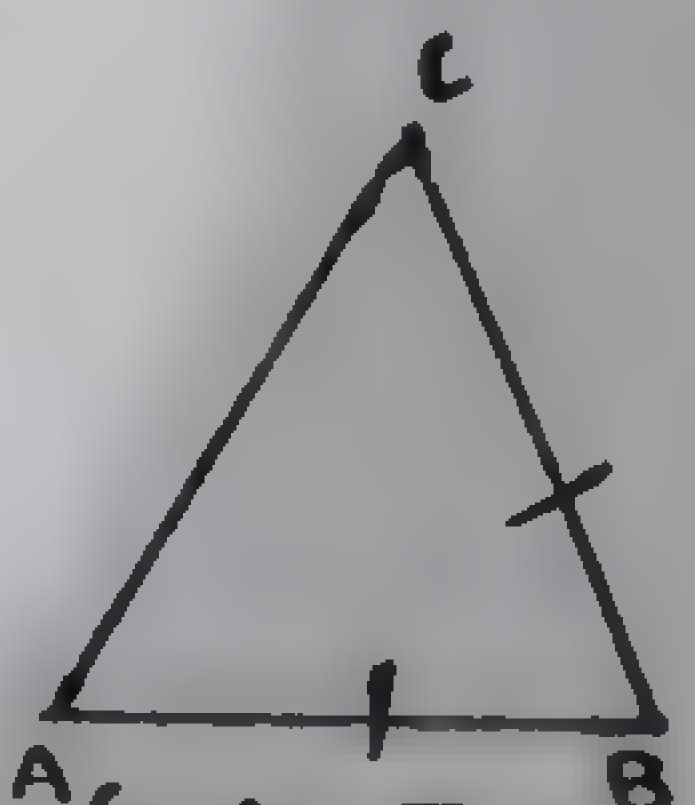
Example:-

Equilateral Triangle

b) Isoscele Triangle:-

A triangle with two sides of equal length is called isoscele triangle.

Example:-

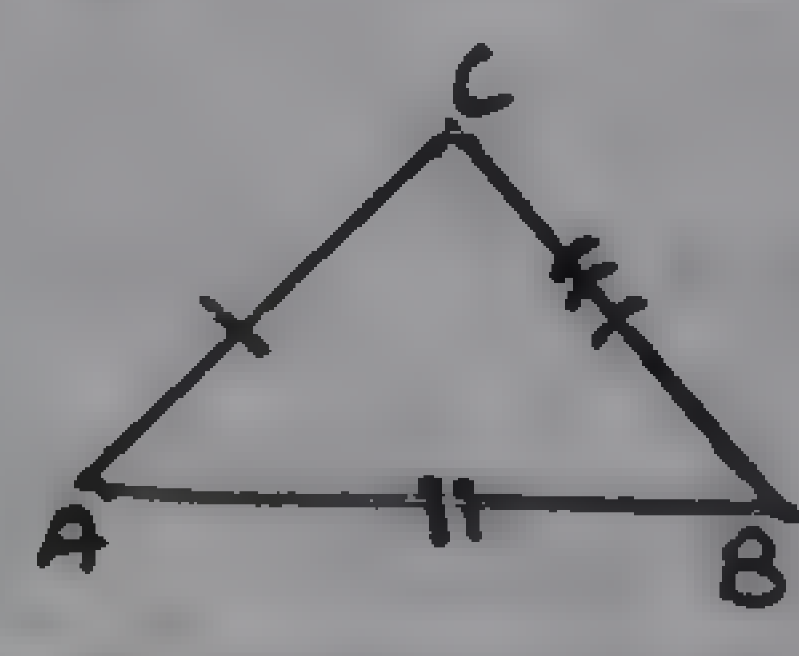


Isoscele Triangle

c) Scalene triangle:-

A triangle with all the sides of different length is called scalene triangle.

Example:-



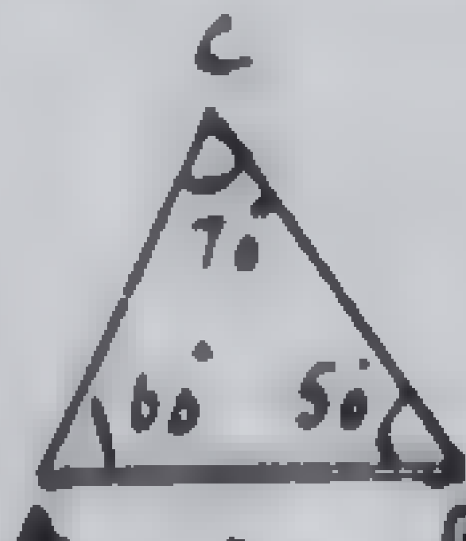
Scalene Triangle

• By Angles:-

a) Acute triangle / Acute-angled triangle:-

A triangle with all the interior angles measuring less than 90° is called acute triangle / acute-angled triangle.

Example:-

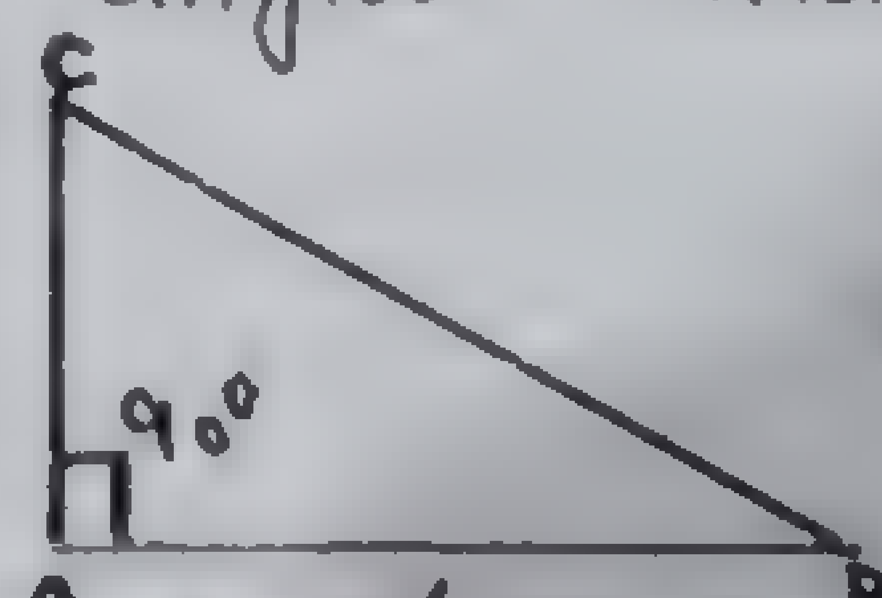


Acute Triangle

b) Right triangle / Right-angled triangle:-

A triangle with one interior angle measuring 90° is called right triangle / right-angled triangle.

Example:-

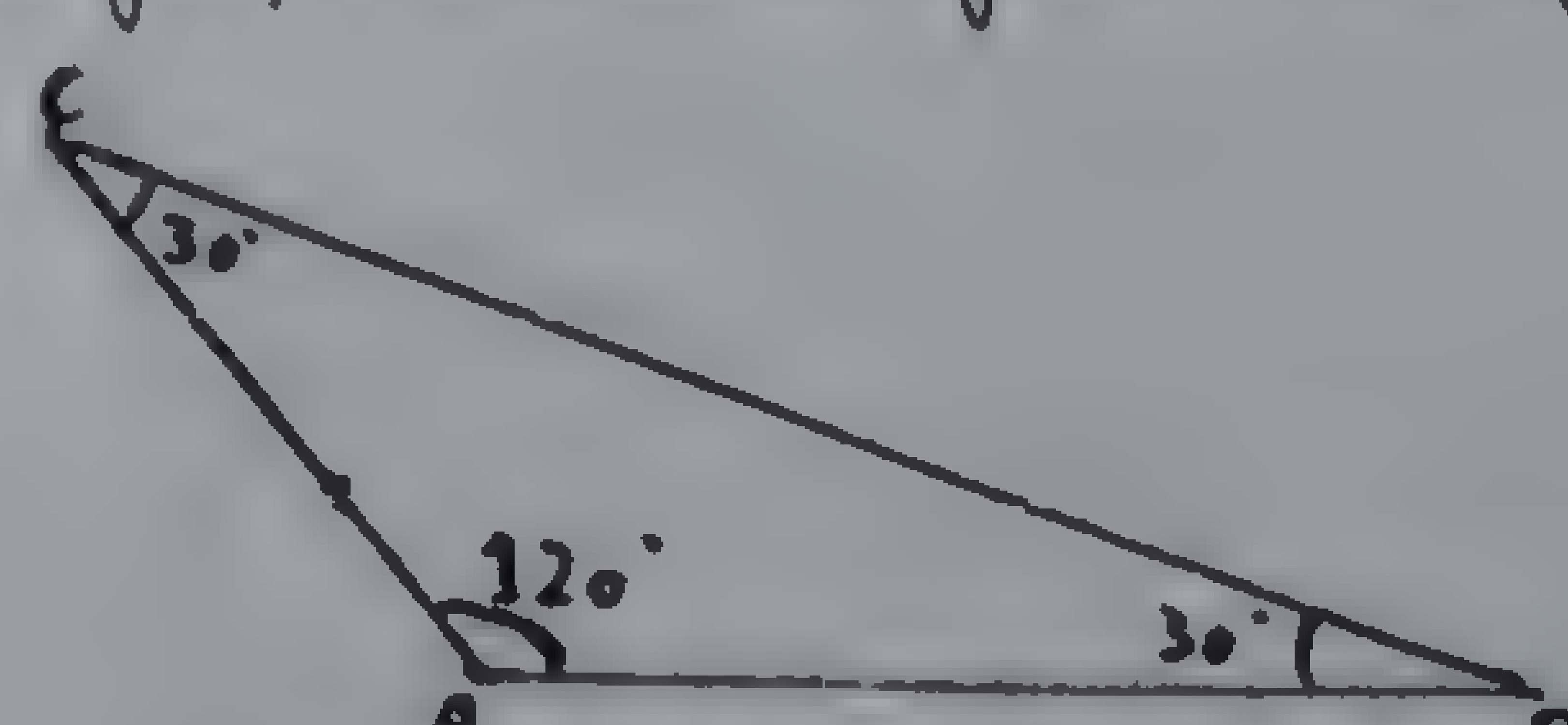


Right Triangle.

c) Obtuse triangle / Obtuse-angled triangle:-

A triangle with one interior angle measuring greater than 90° is called obtuse triangle / obtuse-angled triangle.

Example:-



Obtuse Triangle.

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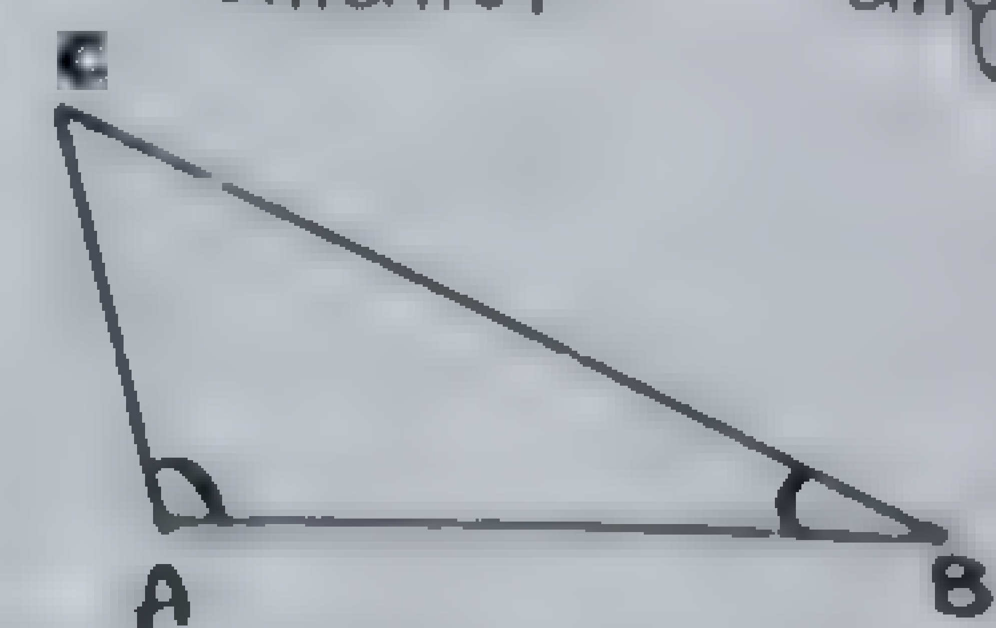
ii) Theorem 13·1·1:-

If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it

iii) Theorem 13-1-2:-

If two angles of a triangle are unequal in measure, the side opposite to the grēter angle is longer than the side opposite to the smaller angle.



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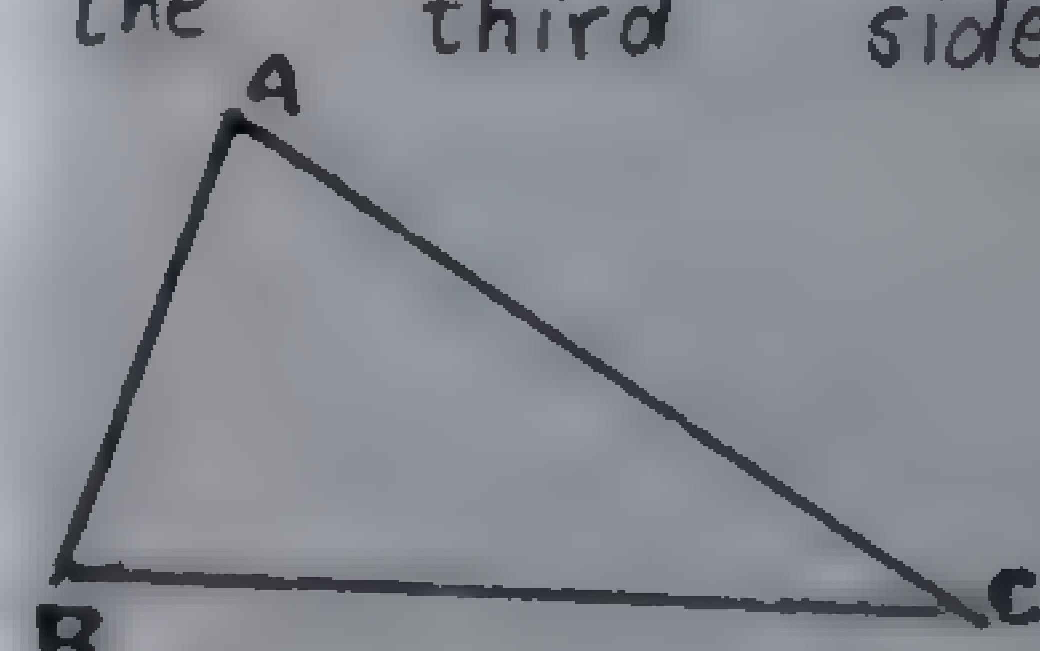
iv) Theorem 13-1-3:-

The sum of the lengths of any two sides of a triangle is greater than the length of. the third side.

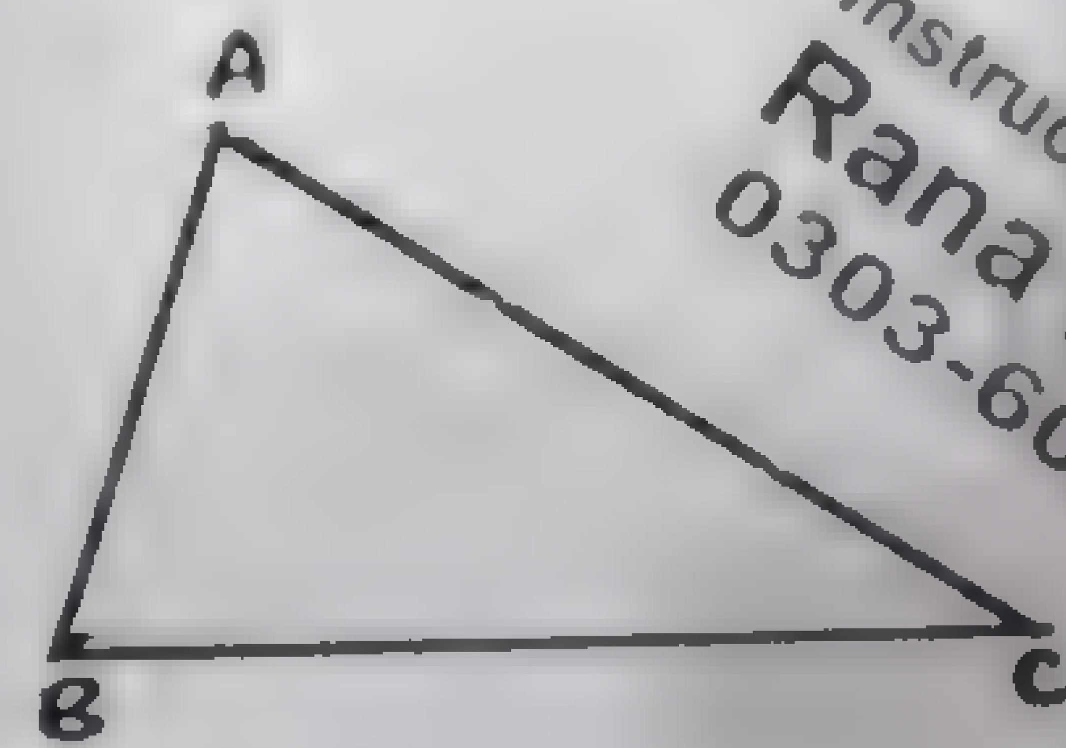
Conditions:-

- $m\overline{AB} + m\overline{AC} > m\overline{BC}$
- $m\overline{AB} + m\overline{BC} > m\overline{AC}$
- $m\overline{BC} + m\overline{AC} > m\overline{AB}$



v) Theorem:-

The difference of measure of two sides of a triangle is less than the measure of third side.



Conditions:-

$m\overline{AC} - m\overline{AB} < m\overline{BC}$

$m\overline{BC} - m\overline{AB} < m\overline{AC}$

$m\overline{BC} - m\overline{AC} < m\overline{AB}$

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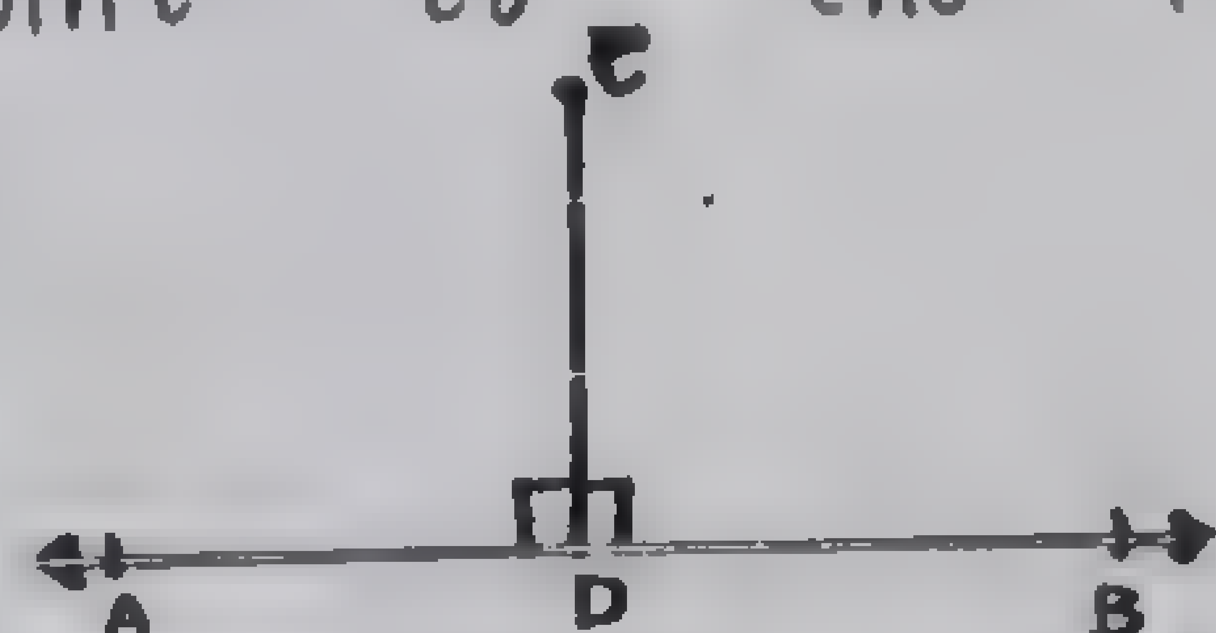
vi) Theorem 13.1.4:-

From a point, outside a line, the perpendicular is the shortest distance from the point to the line.



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vii) Corollaries:-

- The hypotenuse of a right-angled triangle is longer than each of the other two sides.
- In an obtuse-angled triangle, the side opposite to the obtuse angle is longer than each of the other two sides.

---

viii) **Note:-**

- The distance between a line and a point not on it, is the length of the perpendicular line segment from the point to the line.
- The distance between a line and a point lying on it is zero.

ix) **Review Ex. 13:-**

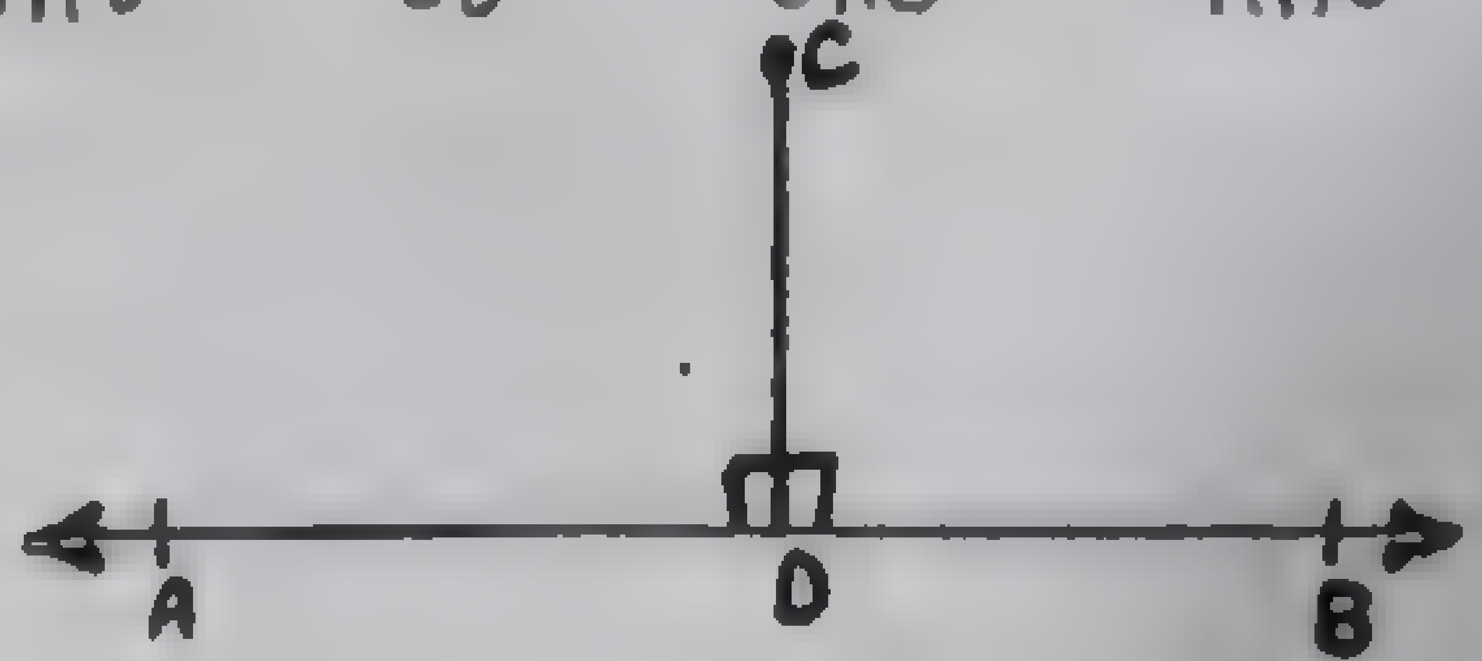
1. Which of the following are true and which are false?

- The angle opposite to the longer side is greater. **True**
- In a right-angled triangle greater angle is 60°. **False**
- In an isosceles right-angled triangle, angles other than right angle are each of 45°. **True**
- A triangle having two congruent sides is called equilateral triangle. **False**
- A perpendicular from a point to the line is shortest distance. **True**
- Perpendicular to line form an angle of 90°. **True**
- A point out side the line is collinear. **False**

- Sum of two sides of triangle is greater than the third. True
- The distance between a line and a point on it is zero. True
- Triangle can be formed of lengths 2cm, 3cm and 5cm. True

2. What will be angle for shortest distance from an outside point to the line?

Ans:- From a point, outside a line, the perpendicular is the shortest distance from the point to the line.



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3. If 13cm, 12cm, and 5cm are the lengths of a triangle, then verify that difference of measure of any two sides of a triangle is less than the measure of third side.

Solution:- Let,

$a = 13cm, \quad b = 12cm, \quad c = 5cm$

Now,

- $a - b < c$

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$13 - 12 = 1 < 5$

- $b - c < a$

  $12 - 5 = 7 < 13$

- $a - c < b$

  $13 - 5 = 8 < 12$

Hence proved, difference of two sides of a triangle is less than the measure of third side.

4. If 10cm, 6cm and 8cm are the lengths of a triangle, then verify that sum of measure of two sides of a triangle is greater then the third side.

Solution:- Let,

$$a = 10cm,\ b = 6cm,\ c = 8cm$$

Now,

- $a + c > b$

  $10 + 8 = 18 > 6$

- $b + c > a$

  $6 + 8 = 14 > 10$

- $a + b > c$

  $10 + 6 = 16 > 8$

Hence proved, the sum of two sides of a triangle is greater than the measure of third side.

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5. 3cm, 4cm, and 7cm are not the lengths of the triangle. Give the reason.

Solution:- Let,

$$a = 3cm,\ b = 4cm,\ c = 7cm$$

Now,

- $a + b > c$

  $3 + 4 = 7 \not> 7$

- $b + c > a$

  $4 + 7 = 11 > 3$

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- $a + c > b$

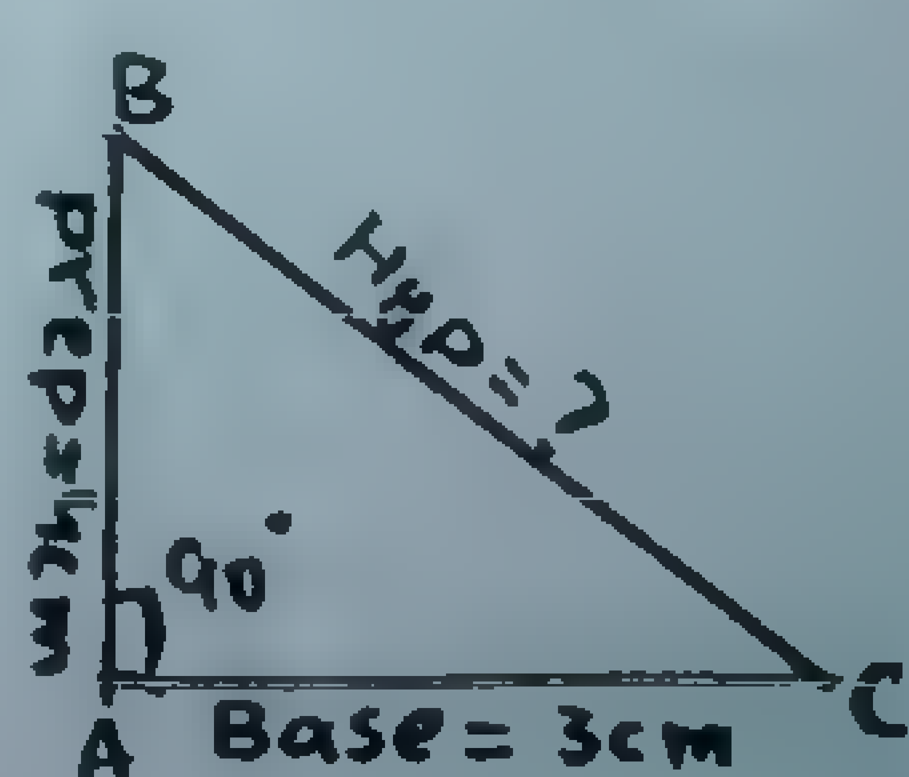
  $3 + 7 = 10 > 4$

As, $a + b \not> c$, So, these are not the lengths of triangle.

6. If 3cm, 4cm are lengths of two sides of a right-angled triangle, then what should be the third length of the triangle. (Hint: Find Hypotenuse).

Solution:- According to Phythagoras theorem,

$$(Hyp)^2 = (Base)^2 + (Prep.)^2$$
$$(Hyp)^2 = (3)^2 + (4)^2$$
$$(Hyp.)^2 = 9 + 16$$
$$(Hyp)^2 = 25$$



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By taking square root on B.S,

$$\sqrt{(Hyp.)^2} = \sqrt{25}$$

$$\boxed{Hyp. = 5\,cm}$$

7. Which of the following sets of lengths can be the lengths of sides of a triangle?

(a) 2cm, 3cm, 5cm **(b) 3cm, 4cm, 5cm**

(c) 2cm, 4cm, 7cm

Solution:-

(a) 2cm, 3cm, 5cm

Let,

$a = 2\,cm$, $b = 3\,cm$, $c = 5\,cm$

Now,

- $a + b > c$

  $2 + 3 = 5 \ngtr 5$

- $b + c > a$

  $3 + 5 = 8 > 2$

- $a + c > b$

  $2 + 5 = 7 > 3$

As, $a + b \ngtr c$, so, these are not lengths of a triangle.

(b) 3cm, 4cm, 5cm

Let,

$a = 3\,cm$, $b = 4\,cm$, $c = 5\,cm$

Now,

- $a+b>c$

  $3+4=7>5$

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- $b+c>a$

  $4+5=9>3$

- $a+c>b$

  $3+5=8>4$

Hence proved, these are the lengths of a triangle.

(c) 2cm, 4cm, 7cm

Let,

$a=2\text{cm},\ b=4\text{cm},\ c=7\text{cm}$

Now,

- $a+b>c$

  $2+4=6\not>7$

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- $b+c>a$

  $4+7=11>2$

- $c+a>b$

  $7+2=9>4$

As, $a+b\not>c$, So, these are not the lengths of a triangle.

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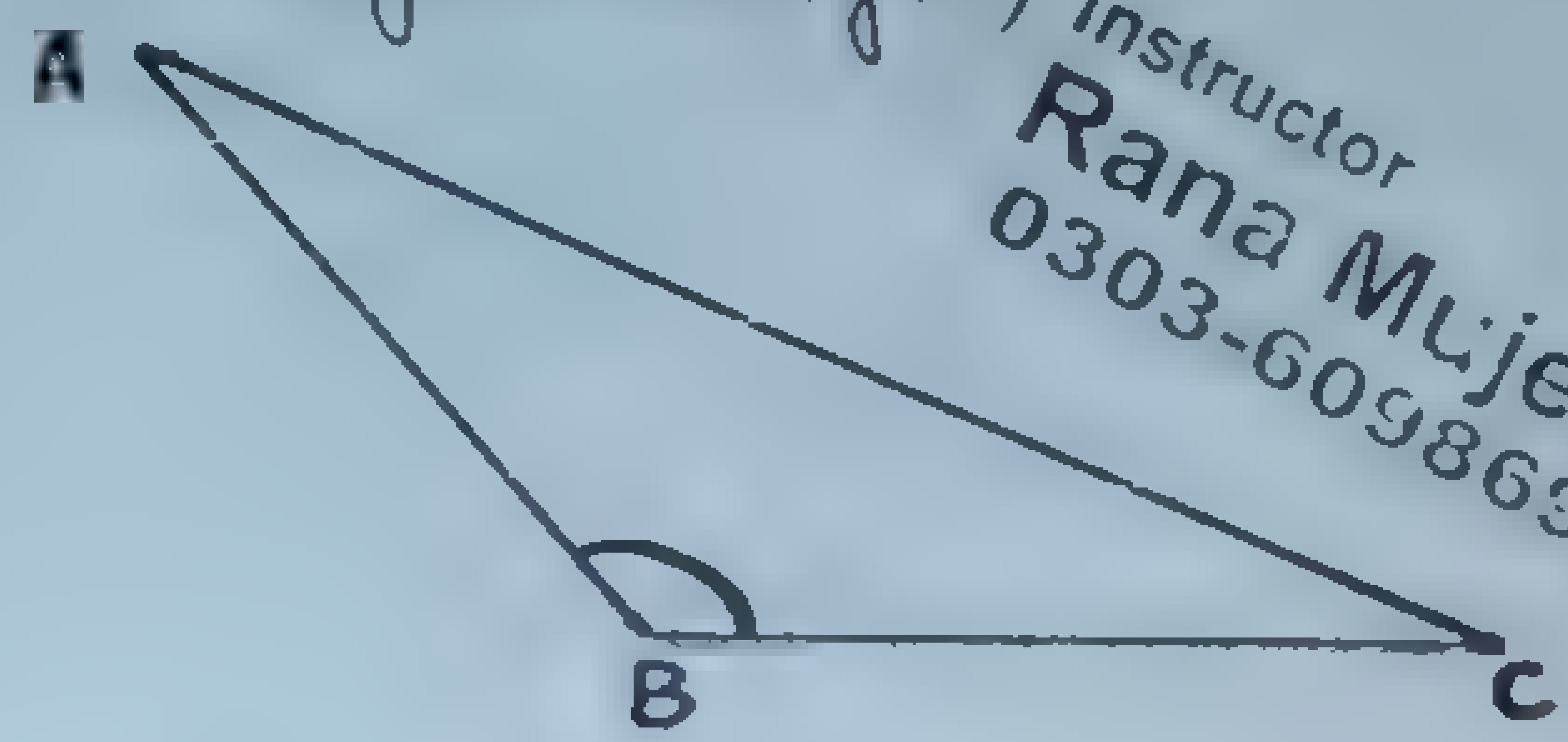
x) Some Important concepts:-

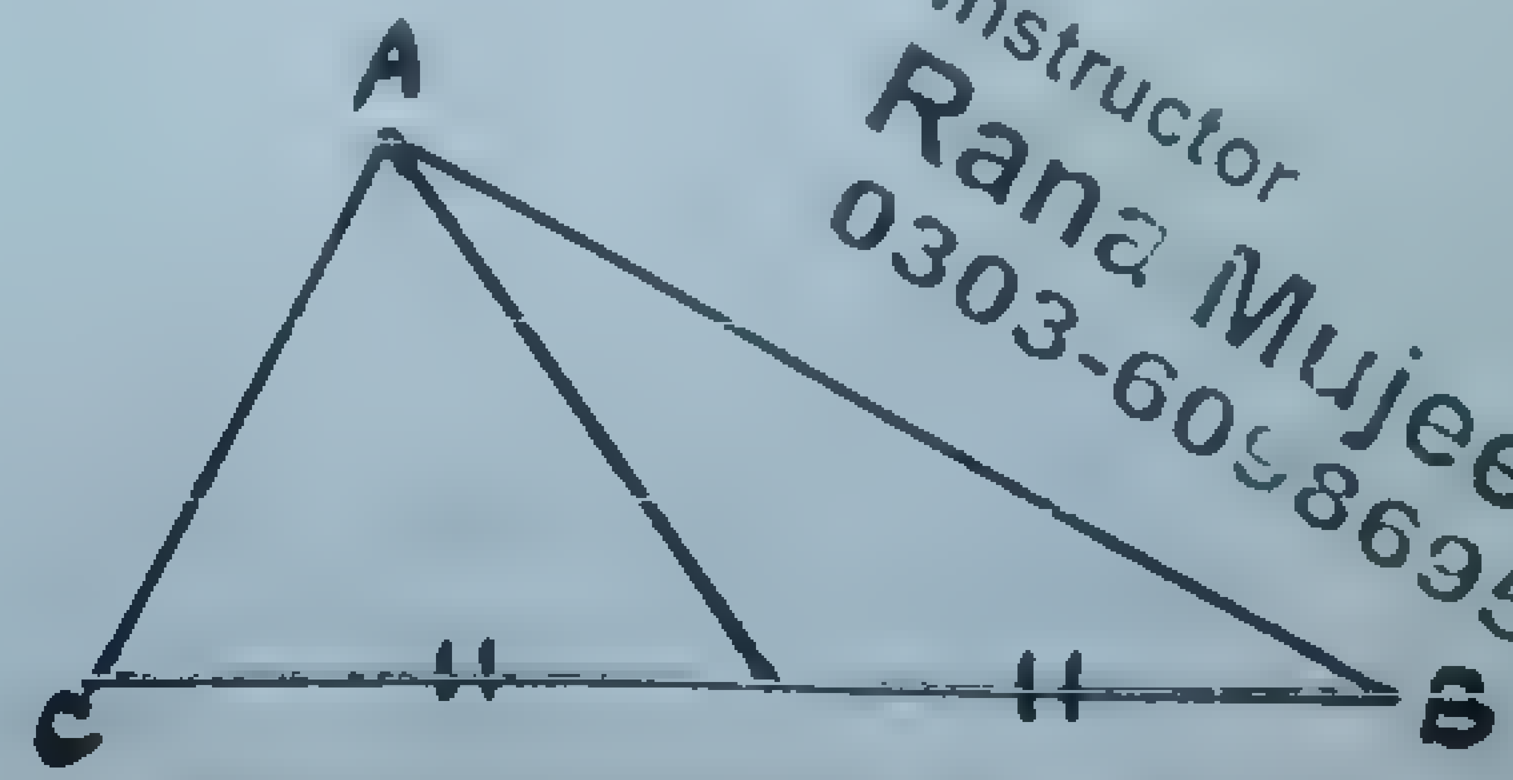
• Important 1:-

In a scalene triangle, the angle opposite to the largest side is of measure greater than 60°. (i.e., two-third of a right-angle).



• Important 2:-

The sum of a measures of two sides of a triangle is greater than twice the measure of the median which bisect the third side.



Chapter # 14:-
" Ratio and Proportion "

Basic Concepts:-

(i) Congruent triangles.
(ii) Similar triangles.

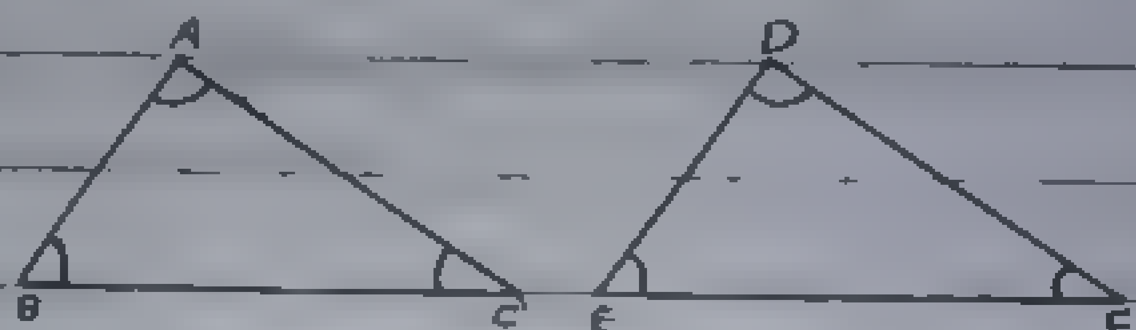
(iii) Ratio.
(iv) Proportion.
(v) 1st and 2nd element of ratio
(vi) Theorem 14.1.1.
(vii) Theorem 14.1.2.
(viii) Theorem 14.1.3.

(ix) Theorem 14.1.4.
(x) Point to be noted.
(xi) Ex14.2 (Q1, (i)(ii)(iii)(iv)(V)).
(xii) Ex14.2 (Q1 & Q2 easily).
(xiii) Review Ex 14.

(i) Congruent Triangles:-

Two triangles are said to be congruent written symbolically as, $\cong$, if there exists a correspondence b/w them, such that all the corresponding sides and angles are congruent. i.e.,

$$\text{If } \begin{cases} \overline{AB} \cong \overline{DE} \\ \overline{BC} \cong \overline{EF} \\ \overline{CA} \cong \overline{FD} \end{cases} \text{ and } \begin{cases} \angle A \cong \angle D \\ \angle B \cong \angle E \\ \angle C \cong \angle F \end{cases}$$



(ii) Similar Triangles:-

Two (or more) triangles are called similar (symbol '$\sim$') if they are equiangular and measure of their corresponding sides are proportional. i.e.,

In $\triangle ABC \longleftrightarrow \triangle DEF$

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle E$ and

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{BC}}{m\overline{EF}} = \frac{m\overline{CA}}{m\overline{FD}}$$

(iii) Ratio:-

We defined ratio $a:b = \frac{a}{b}$ as the comparision of two alike quantities a and b called the terms of a ratio.

e.g.,

2:3 , 3:5 , etc.

(iv) Proportion:-

Equality of two ratios is defined as proportion. i.e, if a:b=c:d , then a, b, c and d are said to be in proportion.

e.g.,

2:3 = 1:4 , etc.

(v) 1st and 2nd element of ratio:-

In a ratio. a:b , a is called 1st element of ratio or antecedant and b is called 2nd element of ratio or consequent.

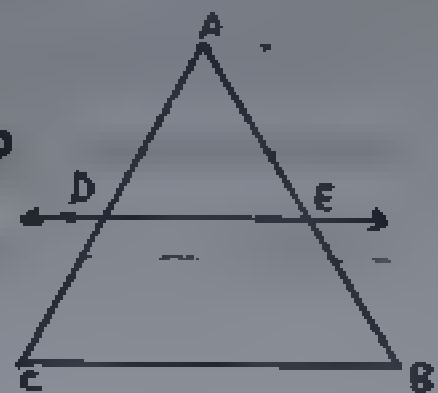
e.g.,

$\boxed{2} : \boxed{3} \rightarrow$ consequent.

antecedant

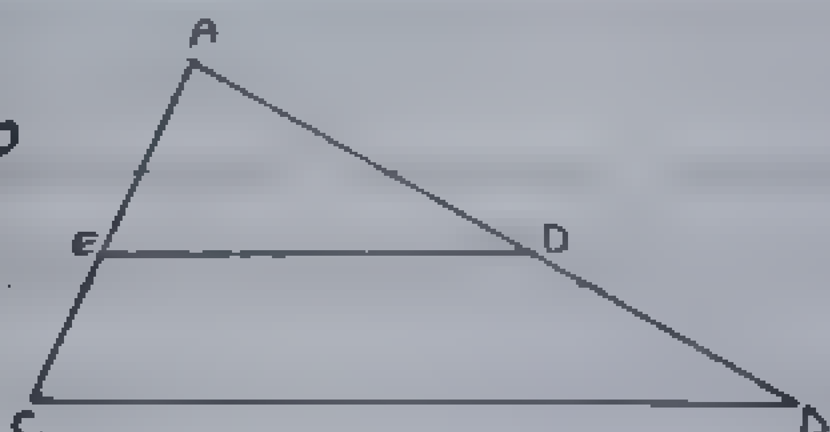
(vi) Theorem 14.1.1:-

A line parallel to one side of a triangle and intersecting the other two sides divides them proportionally.



(vii) Theorem 14.1.2:-

If a line segment intersects the two sides of a triangle in the same ratio, then it is parallel to the third side.



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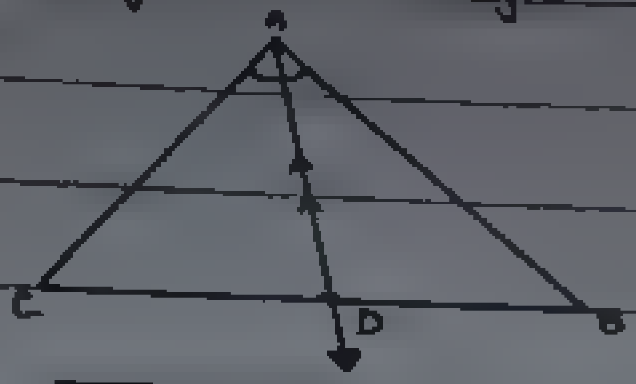
(viii) Theorem 14.1.3:-

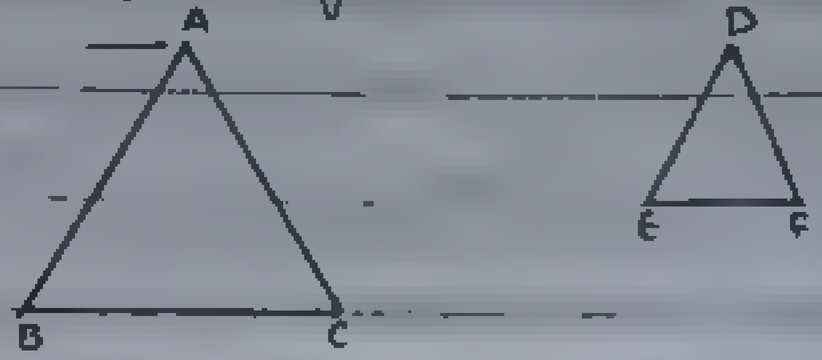
The internal bisector of an angle of a triangle divides the side opposite to it in the ratio of the lengths of the

sides containing the angle.



(ix) Theorem 14.1.4:-

If two triangles are similar, then the measures of their corresponding sides are proportional.



$\Delta ABC \sim \Delta DEF$

i.e., $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

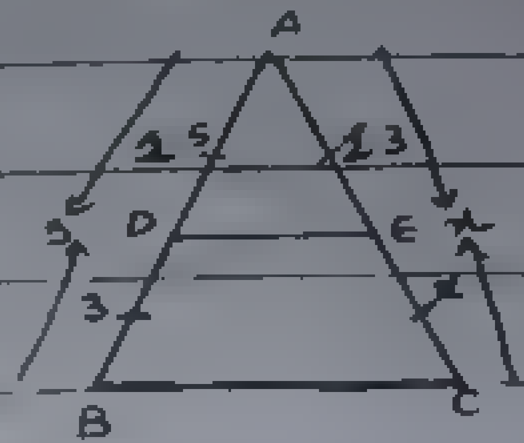
(x) Point to be Noted:-

- Two points determine a line and three non-collinear points determine a plane.
- A line segment has exactly one midpoint.
- If two intersecting lines form equal adjacent angles, the lines are perpendicular.
- Two congruent triangles are similar also. But two similar triangles are not necessarily congruent, as congruence of their corresponding sides is not necessary.

(xi) Ex 14.1

Q1:- In $\triangle ABC$, $\overline{DE} \parallel \overline{BC}$,

(i) If $m\overline{AD} = 1.5cm$, $m\overline{BD} = 3cm$, $m\overline{AE} = 1.3cm$ then find $m\overline{CE}$.



Let,

$m\overline{CE} = x$

Now,

$mAD : mBD = m\overline{AE} : m\overline{CE}$

$1.5 : 3 = 1.3 : x$

$\frac{1.5}{3} = \frac{1.3}{x}$

$1.3 \times 3 = 1.5x$

$3.9 = 1.5x$

$\frac{3.9}{1.5} = x$

$\boxed{x = 2.6cm}$

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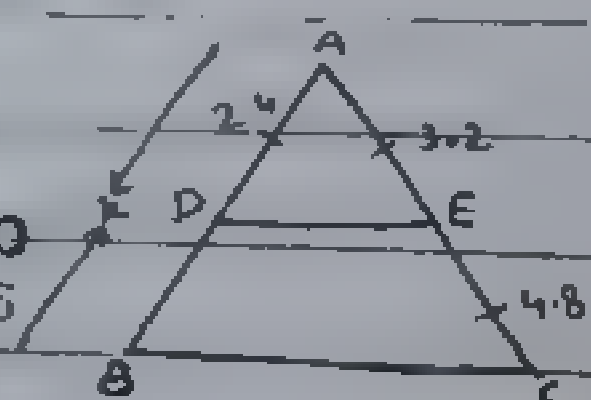
(ii) If $m\overline{AD} = 2.4cm$, $m\overline{AE} = 3.2cm$, $m\overline{EC} = 4.8cm$ then find $m\overline{AB}$.



Let,

$m\overline{AB} = x$

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Now,

$m\overline{AD} : m\overline{AE} = m\overline{DB} : m\overline{EC}$

$2.4 : 3.2 = m\overline{DB} : 4.8$

$\frac{2.4}{3.2} = \frac{m\overline{DB}}{4.8}$

$2.4 \times 4.8 = m\overline{BD} \times 3.2$

$11.52 = m\overline{BD} \times 3.2$

$\dfrac{11.52}{3.2} = m\overline{BD}$

$\boxed{m\overline{BD} = 3.6cm}$

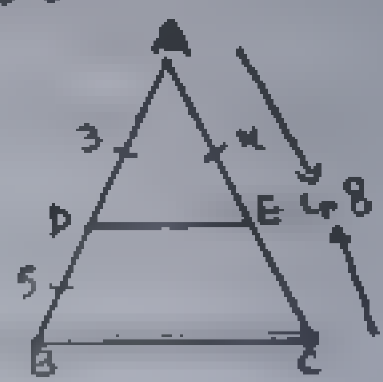
So,

$m\overline{AB} = m\overline{AD} + m\overline{DB}$

$x = 2.4 + 3.6$

$\boxed{x = 6cm}$

(iv) If $\dfrac{m\overline{AD}}{m\overline{DB}} = \dfrac{3}{5}$ and $m\overline{AC} = 4.8cm$ than find $m\overline{AE}$.



Let,

$m\overline{AE} = x$

Now,

$m\overline{AD} : m\overline{DB} = m\overline{AE} : m\overline{EC}$

$3 : 5 = x : m\overline{AC} - m\overline{AE}$

$\dfrac{3}{5} = x : 4.8 - x$

$\dfrac{3}{5} = \dfrac{x}{4.8 - x}$

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$3(4.8 - x) = 5x$

$14.4 - 3x = 5x$

$14.4 = 3x + 5x$

$14.4 = 8x$

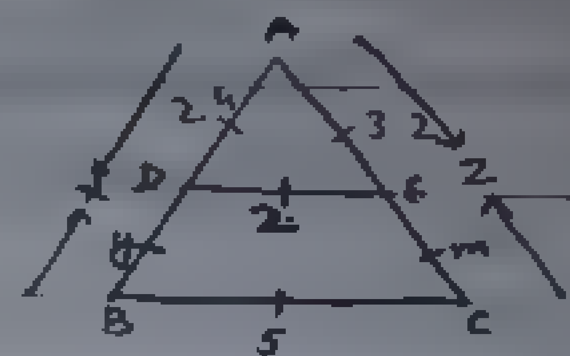
$x = \dfrac{14.4}{8}$

$\boxed{x = 1.8\,cm}$

(iv) If $m\overline{AD} = 2.4\,cm$, $m\overline{AE} = 3.2\,cm$, $m\overline{DE} = 2\,cm$, $mBC = 5\,cm$ than find $m\overline{AB}$.

Let,

$m\overline{AB} = x$, $m\overline{DB} = y$, $m\overline{AC} = z$, $m\overline{CE} = m$.



Now,

$\triangle ADE \sim \triangle ABC$

$$\frac{m\overline{AD}}{m\overline{AB}} = \frac{m\overline{DE}}{m\overline{BC}} = \frac{m\overline{AE}}{m\overline{AC}}$$

$$\frac{2.4}{x} = \frac{2}{5} = \frac{3.2}{z}$$

$\frac{2.4}{x} = \frac{2}{5}$ and $\frac{2}{5} = \frac{3.2}{z}$

$2.4 \times 5 = 2x$ and $2 \times z = 3.2 \times 5$

$12 = 2x$ and $2z = 16$

$\frac{\cancel{12}\,6}{\cancel{2}\,1} = x$ and $z = \frac{\cancel{16}\,8}{\cancel{2}}$

$\boxed{x = 6\,cm}$ and $\boxed{z = 8\,cm}$

Here,

$y = m\overline{AB} - m\overline{AD}$

$y = 6 - 2.4$

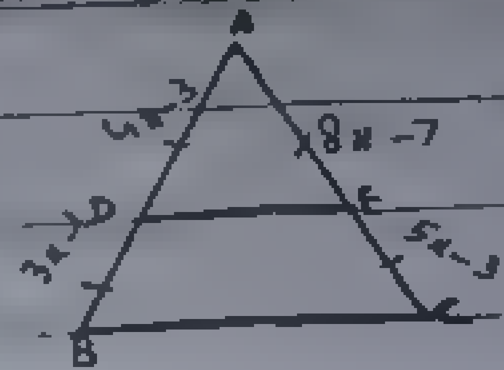
$\boxed{y = 3.6\,cm}$

$m = m\overline{AC} - m\overline{AE}$

$m = 8 - 3.2$

$\boxed{m = 4.8\text{ cm}}$

(v) If $m\overline{AD} = 4x-3$, $m\overline{AE} = 8x-7$, $m\overline{BD} = 3x-1$ and $m\overline{CE} = 5x-3$, find value of x.



As,

$m\overline{AD} : m\overline{AE} = m\overline{DB} : m\overline{CE}$

$4x-3 : 8x-7 = 3x-1 : 5x-3$

$\frac{4x-3}{8x-7} = \frac{3x-1}{5x-3}$

$(4x-3)(5x-3) = (8x-7)(3x-1)$

$20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$

$20x^2 - 27x + 9 = 24x^2 - 29x + 7$

$0 = 24x^2 - 29x + 7 - 20x^2 + 27x - 9$

$0 = 4x^2 - 2x - 2$

$4x^2 - 2x - 2 = 0$

$4x^2 - 4x + 2x - 2 = 0$

$4x(x-1) + 2(x-1) = 0$

$(x-1)(4x+2) = 0$

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$x - 1 = 0$

$\boxed{x = 1}$

$4x + 2 = 0$

$4x = -2$

$x = -\cancel{2}^{1}/\cancel{4}_{2}$

$\boxed{x = -\frac{1}{2}}$

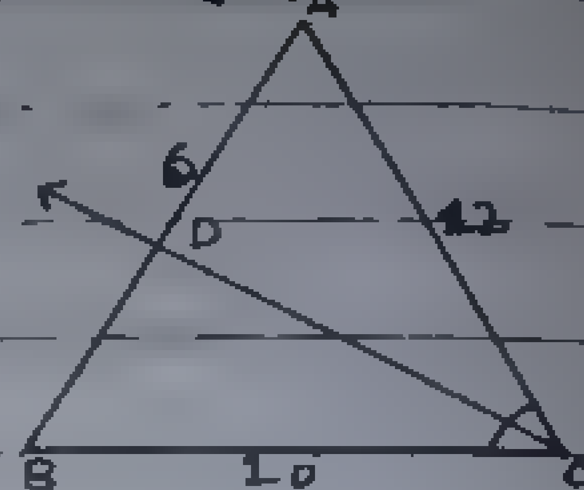
$x = -\frac{1}{2}$ is not possible

so, $\boxed{x = 1}$

(xii) Ex 14.2:-

Q1:- In ΔABC as shown in the figure, $\overrightarrow{CD}$ bisects $\angle C$ and meets $\overline{AB}$ at D. is equal to

(a) 5 (b) 16 (c) 10 (d) 18



Let,

$m\overline{BD} = x$

Now,

$m\overline{AD} : m\overline{BD} = m\overline{AC} : m\overline{BC}$

$6 : x = 12 : 10$

$\frac{6}{x} = \frac{12}{10}$

$6 \times 10 = 12x$

$60 = 12x$

$\frac{\cancel{60}^{5}}{\cancel{12}_{1}} = x$

$\boxed{x = 5cm}$

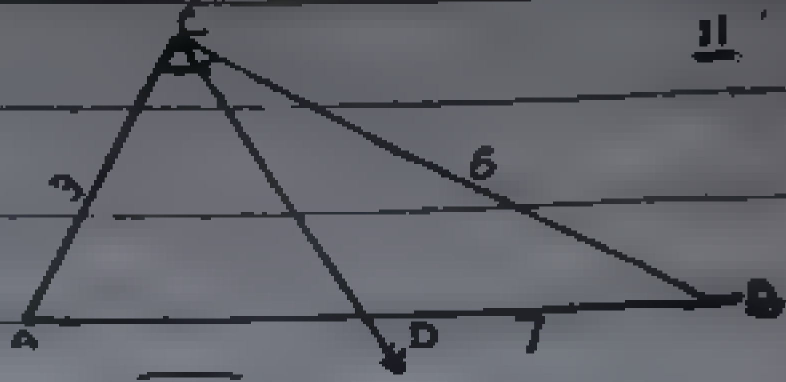
So,

$m\overline{BD}$ is equal to option '(a)'.

Q2:- In ΔABC shown in the figure, $\overrightarrow{CD}$ bisects $\angle C$. If $m\overline{AC} = 3$, $m\overline{CB} = 6$ and $m\overline{AB} = 7$, then find $m\overline{AD}$ and $m\overline{DB}$.

Let,

$m\overline{AD} = x$



Now,

$m\overline{AD} : m\overline{DB} = m\overline{AC} : m\overline{CB}$

$x : 7 - x = 3 : 6$

$\frac{x}{7-x} = \frac{3}{6}$

$6x = 3(7x - x)$

$6x = 21x - 3x$

$6x + 3x = 21$

$9x = 21$

$x = \frac{\cancel{21}^{7}}{\cancel{9}_{3}}$

$x = \frac{7}{3}$

$\boxed{m\overline{AD} = \frac{7}{3}}$

$m\overline{DB} = 7 - \frac{7}{3}$

$m\overline{DB} = \frac{21-7}{3}$

$\boxed{m\overline{DB} = \frac{14}{3}}$

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(xiii) Review Ex 14

Q 1:– Which one of the following are true and which are false?

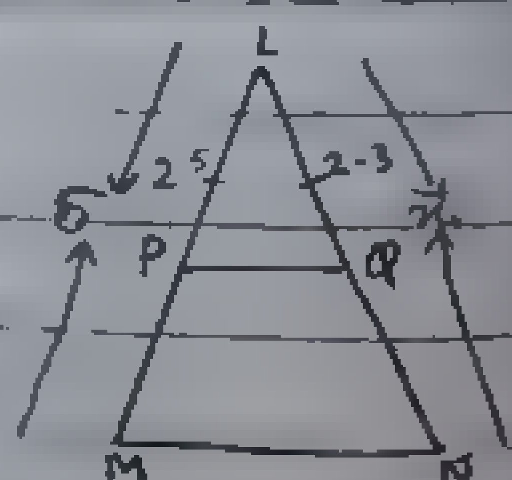
- Congruent triangles are of same

size and shape. True

- Similar triangles are of same shape but different sizes. True
- Symbol used for congruent is '≅'. True
- Symbol used for similarity is '~'. True
- Congruent triangles are similar. True
- Similar triangles are congruent. False
- A line segment has only one mid-point. True
- One and only one line can be drawn through two points. True
- Proportion is non-equality of two ratios. False
- Ratio has no unit True

Q3:- In $\triangle LMN$ shown in figure $\overline{MN} \parallel \overline{PQ}$.

(i) If $m\overline{LM} = 5cm$, $m\overline{LP} = 2.5cm$, $m\overline{LQ} = 2.3cm$ then find $m\overline{LN}$



Let,

$m\overline{LN} = x$

Now,

$m\overline{LM} : m\overline{LP} = m\overline{LN} : m\overline{LQ}$

$5 : 2.5 = x : 2.3$

$\frac{5}{2.5} = \frac{x}{2.3}$

$5 \times 2.3 = 2.5x$

$11.5 = 2.5x$

$\frac{11.5}{2.5} = x$

$\boxed{x = 4.6\,cm}$

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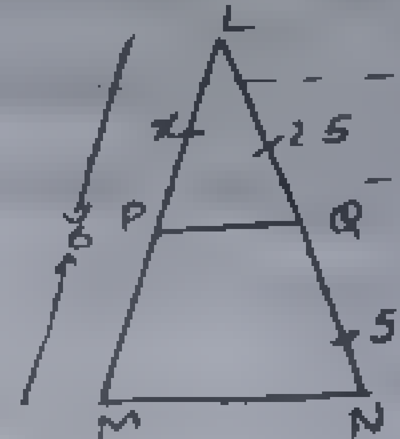
Q If $m\overline{LM} = 6cm$, $m\overline{LQ} = 2.5cm$, $mQN = 5cm$ then find $m\overline{LP}$.



Let,

$m\overline{LP} = x$

Now,

$m\overline{LQ} : m\overline{LP} = m\overline{LN} : m\overline{LQ}$

$6 : x = m\overline{LQ} + m\overline{QN} : 2.5$

$6 : x = 2.5 + 5 : 2.5$

$6 : x = 7.5 : 2.5$

$\frac{6}{x} = \frac{7.5}{2.5}$

$6 \times 2.5 = 7.5x$

$15 = 7.5x$

$\frac{\cancel{15}^{2}}{\cancel{7.5}_{1}} = x$

$\boxed{x = 2\,cm}$

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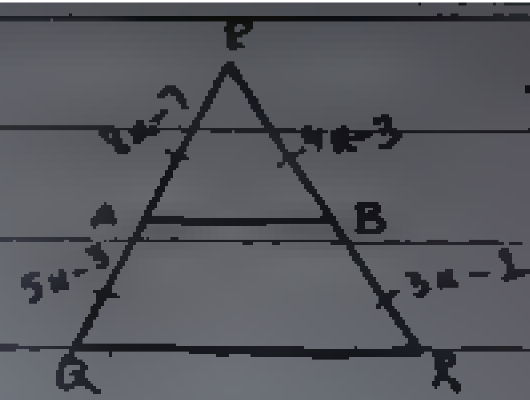
Q4:- In the shown figure, Let $m\overline{PA} = 8x-7$, $m\overline{PB} = 4x-3$, $mAQ = 5x-3$, $m\overline{BR} = 3x-1$, then find value of x

if $\overline{AB} \parallel \overline{QR}$.



As,

$$m\overline{PA} : m\overline{PB} = m\overline{AQ} : m\overline{BR}$$

$$8x-7 : 4x-3 = 5x-3 : 3x-1$$

$$\frac{8x-7}{4x-3} = \frac{5x-3}{3x-1}$$

$$(8x-7)(3x-1) = (4x-3)(5x-3)$$

$$24x^2-8x-21x+7 = 20x^2-12x-15x+9$$

$$24x^2-29x+7 = 20x^2-27x+9$$

$$24x^2-29x+7-20x^2+27x-9 = 0$$

$$4x^2-2x-2 = 0$$

$$4x^2-4x+2x-2 = 0$$

$$4x(x-1)+2(x-1) = 0$$

$$(x-1)(4x+2) = 0$$

$$x-1 = 0$$

$$\boxed{x = 1}$$

$$4x+2 = 0$$

$$4x = -2$$

$$x = -\frac{2}{4}$$

$$\boxed{x = -\frac{1}{2}}$$

$x = -\frac{1}{2}$ is not possible

So

$$\boxed{x = 1}.$$

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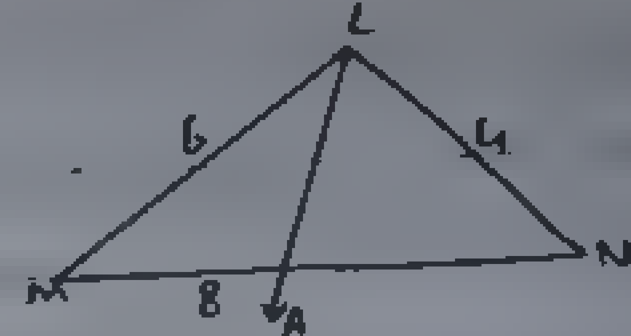
Q5:- In ΔLMN shown in figure $\overrightarrow{LA}$ bisects $\angle L$. If $m\overline{LN} = 4$, $m\overline{LM} = 6$, $m\overline{MN} = 8$, then find $m\overline{MA}$ and $m\overline{AN}$.



Let

$m\overline{MA} = x$

Now,

$$m\overline{MA} : m\overline{AN} = m\overline{ML} : m\overline{LN}$$

$$x : 8 - x = 6 : 4$$

$$\frac{x}{8-x} = \frac{6}{4}$$

$$4x = 6(8 - x)$$

$$4x = 48 - 6x$$

$$4x + 6x = 48$$

$$10x = 48$$

$$x = \frac{48}{10}$$

$$x = 4.8$$

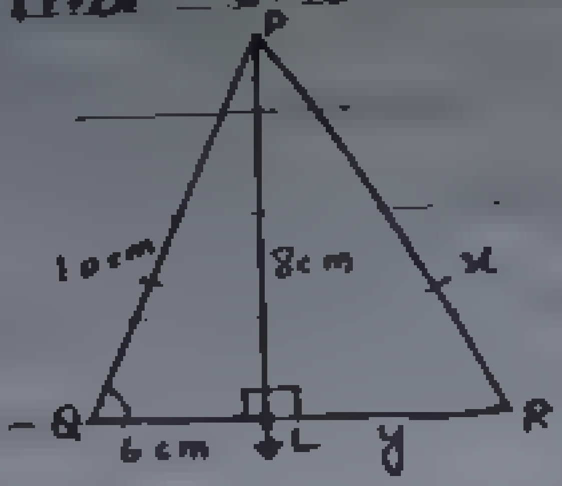
$$\boxed{mMA = 4.8}$$

$$m\overline{AN} = mMN - m\overline{MA}$$

$$m\overline{AN} = 8 - 4.8$$

$$\boxed{m\overline{AN} = 3.2}$$

Q6:- In Isosceles ΔPQR shown in figure, find the value of x and y.



اس سوال کو حل کرنے کے دو طریقے ہیں۔

- **Method I:-**

Statements	Reasons
$m\overline{PR} = m\overline{PQ}$	$\because$ Isosceles triangle
$\boxed{x = 10cm}$	
Now,	
$\Delta PQL \leftrightarrow \Delta PLR$	
$m\overline{PQ} \cong m\overline{PR}$	Hypotenuse
$m\overline{PL} \cong m\overline{PL}$	Common
$\Delta PQL \cong \Delta PLR$	H.S $\cong$ H.S
So	
$\boxed{y = 6cm}$	

- **Method II:-**

$m\overline{PR} = mPQ$ $\quad\because$ Isosceles triangles

$\boxed{x = 10cm}$

In ΔPQL,

$(Hyp)^2 = (Base)^2 + (Perp)^2$

$(10)^2 = (6)^2 + (Perp)^2$

$$100 = 36 + (Prep)^2$$

$$100 - 36 = (Prep)^2$$

$$\sqrt{64} = \sqrt{(Prep)^2}$$

$$\boxed{Prep = 8cm}$$

In ΔPLR,

$$(Hyp)^2 = (Base)^2 + (Perp)^2$$

$$(10)^2 = (y)^2 + (8)^2$$

$$100 = y^2 + 64$$

$$100 - 64 = y^2$$

$$\sqrt{36} = \sqrt{y^2}$$

$$\boxed{y = 6cm}$$

1

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Chapter No 15:-
"Phythagoras theorem"

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Basic Concepts:-

(i) Right-angled triangle.
(ii) Phythagoras theorem.
(iii) Converse of phythagoras theorem.
(iv) Corollary.
(v) Ex 15(Q1. Q2. Q3. Q6(ii), Q7. Q8
(vi) Review Exercise 15

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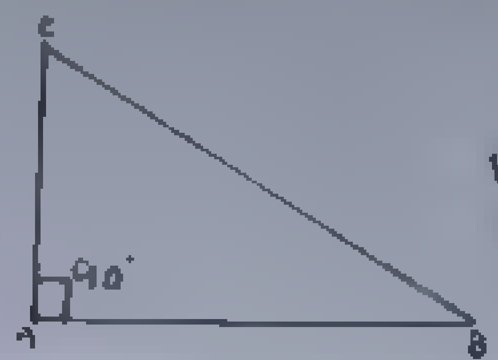
(i) Right-angled triangle:-

A triangle with one interior angle is measuring 90° is called right-angled triangle.

e.g.,



(ii) Phythagoras theorem:-

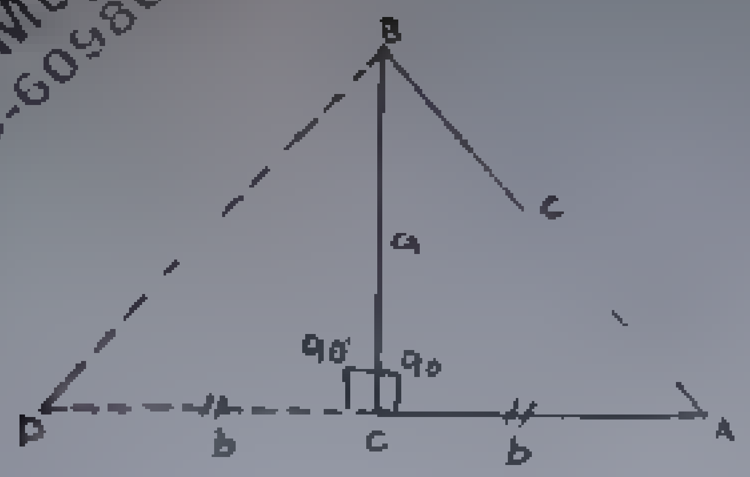
In a right-angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of other two sides.

Formula:-

$$c^2 = a^2 + b^2$$

(iii) Converse of phythagoras theorem:-

If the square of on side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right angled triangle.



Formula:-

$$a^2+b^2=c^2$$

(iv) Corollary:-

Let "c" be the longest side "a, b and c" of a triangle.

- If $a^2+b^2=c^2$, then trianale is right.
- If $a^2+b^2>c^2$, then triangle is acute.
- If $a^2+b^2<c^2$, then triangle is obtuse.

(v) Ex. 15

1- Verify that the Δs having the following measures of sides are right-angled.

(i) $a=5cm.$ $b=12cm.$ $c=13cm$

According to phythagoras theorem,

$c^2=a^2+b^2$

$(13)^2=(5)^2+(12)^2$

$169=25+144$

169 = 169

Hence, it is a right-angled triangle.

(ii) a = 1.5cm, b = 2cm, c = 2.5cm

According to phythagoras theorem,

$c^2 = a^2 + b^2$

$(2.5)^2 = (1.5)^2 + (2)^2$

$6.25 = 2.25 + 4$

$6.25 = 6.25$

Hence, it is a right-angled triangle

(iii) a = 9cm, b = 12cm, c = 15 cm

According to phythagoras theorem,

$c^2 = a^2 + b^2$

$(15)^2 = (9)^2 + (12)^2$

$225 = 81 + 144$

$225 = 225$

Hence, it is a right-angled triangle.

(iv) a = 16cm, b = 30cm, c = 34cm

According to phythagoras theorem,

$c^2 = a^2 + b^2$

$(34)^2 = (16)^2 + (30)^2$

$1156 = 256 + 900$

$1156 = 1156$

Hence, it is a right-angled triangle.

2:- Here,

Hyp. = $c = a^2 + b^2$

Perp. = $b = 2ab$

Base = $a = a^2 - b^2$

According to phythagoras theorem,

$$c^2 = a^2 + b^2$$

$$(a^2+b^2)^2 = (a^2-b^2)^2 + (2ab)^2$$

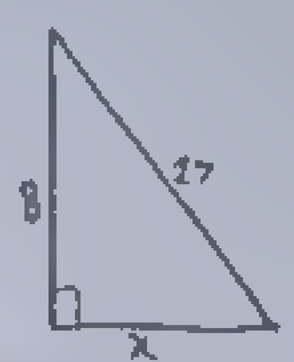
$$(a^2)^2 + (b^2)^2 + 2(a^2)(b^2) = (a^2)^2 + (b^2)^2 - 2(a^2)(b^2) + 4a^2b^2$$

$$a^4 + b^4 + 2a^2b^2 = a^4 + b^4 - 2a^2b^2 + 4a^2b^2$$

$$a^4 + b^4 + 2a^2b^2 = a^4 + b^4 + 2a^2b^2$$

Hence, it is a right-angled triangle.

3:- Here,



Base = $a = x$

Prep. = $b = 8$

Hyp. = $c = 17$

By phythagoras theorem,

$$c^2 = a^2 + b^2$$

$$(17)^2 = (x)^2 + (8)^2$$

$$289 = x^2 + 64$$

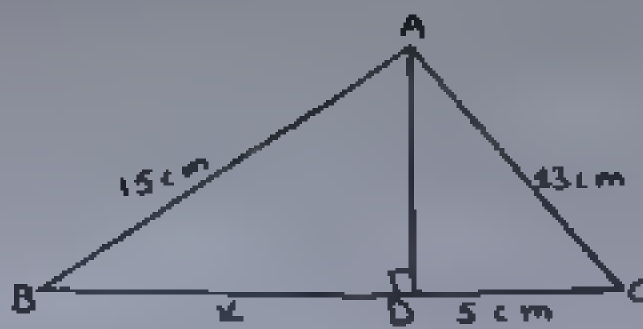
$$289 - 64 = x^2$$

$$\sqrt{225} = \sqrt{x^2}$$

$$\boxed{x = 15}$$

Q6:-

(ii) Find value of "x"?



In $\triangle ADC$,

Base = a = 5cm

prep = b = ?

Hyp = c = 13 cm

By phythagoras theorem,

$$c^2 = a^2 + b^2$$

$$(13)^2 = (5)^2 + b^2$$

$$169 = 25 + b^2$$

$$169 - 25 = b^2$$

$$\sqrt{144} = \sqrt{b^2}$$

$$\boxed{b = 12\,cm}$$

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In $\triangle ABD$,

Base = a = x

Prep = b = 12cm

Hyp = c = 15 cm

By phythagoras theorem,

$$c^2 = a^2 + b^2$$

$$(15)^2 = (x)^2 + (12)^2$$

$$225 = x^2 + 144$$

$$225 - 144 = x^2$$

$$\sqrt{81} = \sqrt{x^2}$$

$$\boxed{x = 9\,cm}$$

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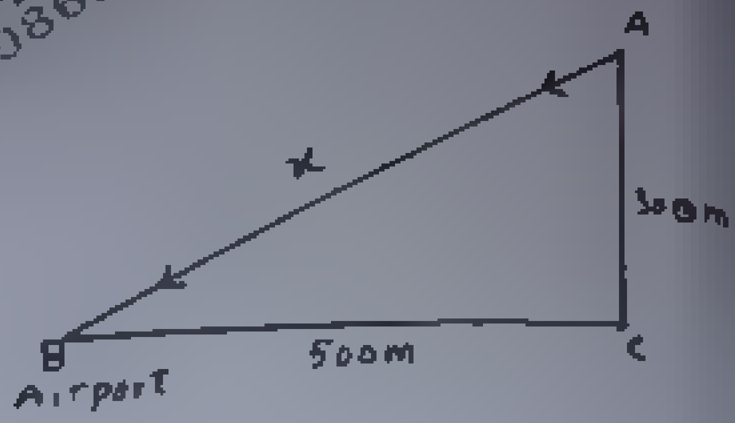
7:-



Here,

Base = a = 500m

Prep = b = 300m

Hyp = c = x

By phythagoras theorem,

$$c^2 = a^2 + b^2$$

$$(x)^2 = (500)^2 + (300)^2$$

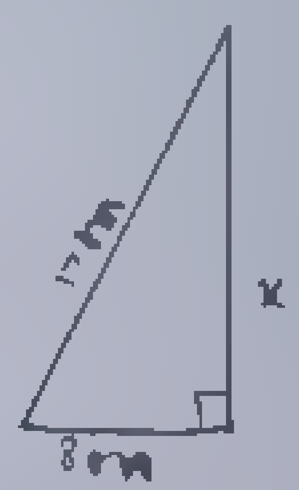
$$x^2 = 250000 + 90000$$

$$\sqrt{x^2} = \sqrt{340000}$$

$$x = \sqrt{34 \times 10000}$$

$$\boxed{x = 100\sqrt{34}}$$

8:- Here,



Base = a = 8m

Prep = b = x

Hyp = c = 17m

By phythagoras theorem,

$$c^2 = a^2 + b^2$$

$$(17)^2 = (8)^2 + (x)^2$$

$$289 = 64 + x^2$$

$$289 - 64 = x^2$$

$$\sqrt{225} = \sqrt{x^2}$$

$$\boxed{x = 15\text{ m}}$$

(vi) Review Exercise 15

1- Which of the following are true and which are not?

(i) In a right-angled triangle greater angle is 90°. True.

(ii) In a right-angled triangle right angle is 60°. False.

(iii) In a right-angled triangle hypotenuse is a side opposite to right angle. True.

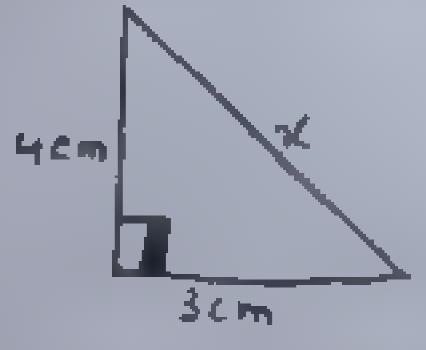
(iv) If a, b, c are sides of right-angled triangle with c as longer side, then $c^2 = a^2 + b^2$. True.

(v) If 3cm and 4cm are two sides of a right-angled triangle, then hypotenuse is 5cm. True

(vi) If hypotenuse of an isosceles right triangle is $\sqrt{2}$ cm, then each of other side is of length 2 cm. False.

2- Find the unknown value in each of the following figures.

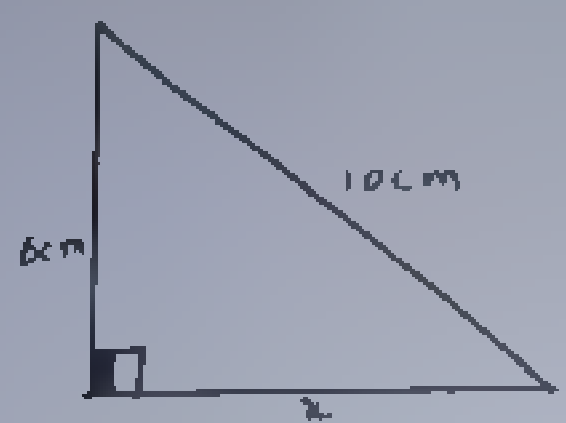
(i)



According to phythagoras theorem,

$$c^2 = a^2 + b^2$$
$$(x)^2 = (3)^2 + (4)^2$$
$$x^2 = 9 + 16$$
$$\sqrt{x^2} = \sqrt{25}$$
$$\boxed{x = 5\text{ cm}}$$

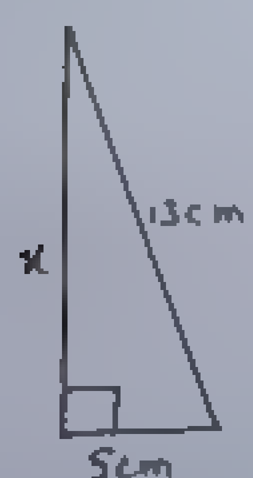
(ii)



According to phythagoras theorem,

$$c^2 = a^2 + b^2$$
$$(10)^2 = (x)^2 + (6)^2$$
$$100 = x^2 + 36$$
$$100 - 36 = x^2$$
$$\sqrt{64} = \sqrt{x^2}$$
$$\boxed{x = 8\text{ cm}}$$

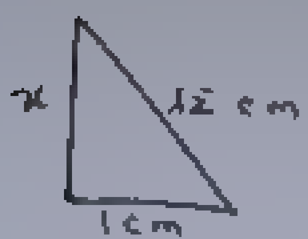
(iii)



According to phythagoras theorem,

$$c^2 = a^2 + b^2$$
$$(13)^2 = (5)^2 + (x)^2$$
$$169 = 25 + x^2$$
$$169 - 25 = x^2$$
$$\sqrt{144} = \sqrt{x^2}$$
$$\boxed{x = 12\ cm}$$

(iv)



According to phythagoras theorem,

$$c^2 = a^2 + b^2$$
$$(\sqrt{2})^2 = (1)^2 + (x)^2$$
$$2 = 1 + x^2$$
$$2 - 1 = x^2$$
$$\sqrt{1} = \sqrt{x^2}$$
$$\boxed{x = 1\ cm}$$

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Chapter # 16

"Theorem Related with Area."

Basic Concepts

(i) Area of a figure.
(ii) Interior of triangle.
(iii) Triangular Region.
(iv) Congruent Area Axiom.
(v) Rectangular Region.
(vi) Interior of Rectangle.
(vii) Altitude of triangle.
(viii) Altited of parallelogram.
(ix) Area of square.
(x) Area of rectangle.
(xi) Area of parallelogram.
(xii) Area of triangle.
(xiii) Important points.
(xiv) Review Exercise 16.

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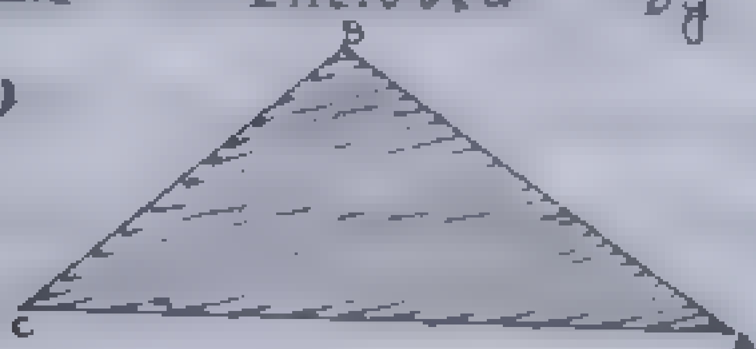
(i) Area of a figure:-

The region enclosed by the bounding lines of a closed figure is called area of a figure. The area of a closed region is expressed in square units (say, sq m, m^2) i.e., a positive real number.

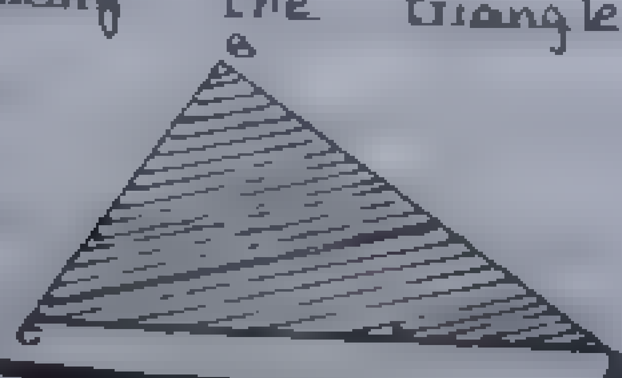
(ii) Interior of triangle:-

The interior of a triangle is the part of the plane enclosed by the triangle. e.g.,



(iii) Triangular Region:-

A triangular region is the union of a triangle and its interior i.e., the three line segments forming the triangle and its interior.



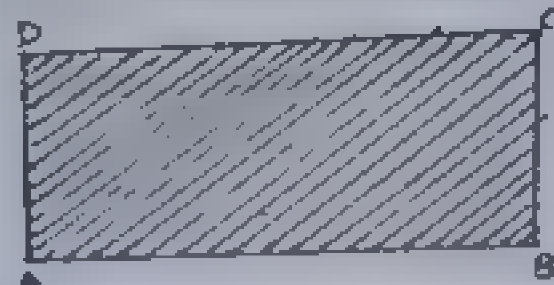
(iv) Congruent Area Axiom:-

If $\Delta ABC \cong \Delta PQR$, then area of (region ΔABC) = area of (region ΔPQR).

(v) Rectangular Region:-

A rectangular region is the union of a rectangle and its interior.

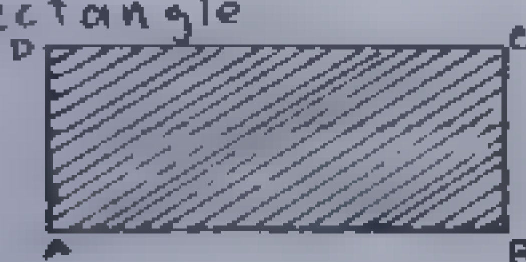
e.g.,



(vi) Interior of Rectangle:-

The interior of a rectangle is the part of the plane enclosed by the rectangle

e.g.,

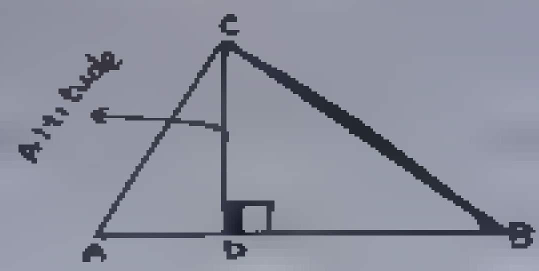


(vii) Altitude of a triangle:-

If one side of a triangle is taken as its base, the perpendicular to that side,

from the opposite vertex is called the altitude or height of a triangle.

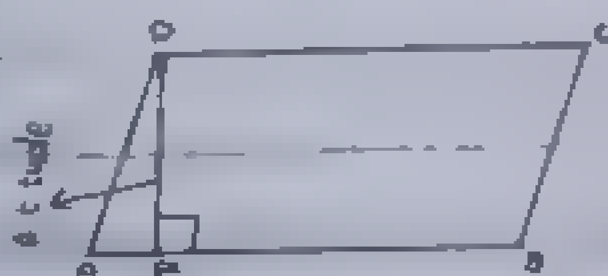
e.g.,



(viii) Altitude of a Parallelogram:-

If one side of a parallelogram is taken as its base, the perpendicular distance between that side and the side parallel to it, is called altitude or height of a parallelogram.

e.g.,



(ix) Area of a square:-

The region enclosed by the bounding lines of a square is called the area of square.

In a square,

Area of a square = Side X Side.

(x) Area of a rectangle:-

The region enclosed by the bounding lines of a rectangle is called the area of rectangle.

In a rectangle,

Area of rectangle = Length X Width.

(xi) Area of a triangle:-

The region enclosed by the bounding lines of a triangle is called the area of triangle

In a triangle,

Area of triangle = $\frac{1}{2}$ X Base X Altitude

(xii) Area of parallelogram:-

The region enclosed by the bounding lines of a parallelogram is called the area of parallelogram.

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In a parallelogram,

Area of parallelogram = Base X Altitude.

(xiii) Important points:—

- Parallelograms on the same base and between the same parallel lines (or of the same altitude) are equal in area.
- Parallelograms on the equal bases and having the same (or equal) altitude are equal in area
- Triangles on the same base and of the same (i e, equal) altitudes are equal in area.
- Triangles on equal bases and of equal altitudes are equal in area.

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(xiv) Review Exercise 16:—

1:— Which one of he following are true and which are false?

(i) Area of a figure means region enclosed by bounding lines of closed figure **True**.

(ii) Similar figures have same area. **False**.

(iii) Congruent figures have same area. **True**.

(iv) A diagonal of a parallelogram divided it into two non-congruent triangles

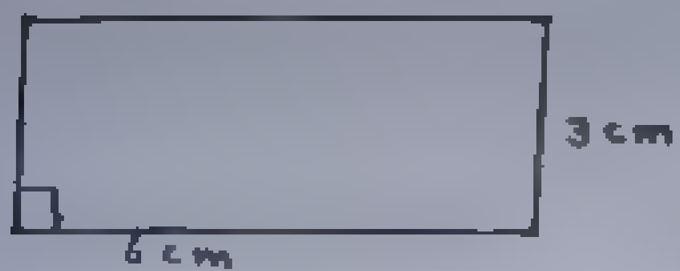
False.

Altitude of a triangle means perpendicular from vertex to the opposide side (base). True.

(vi) Area of a parallelogram is equal to the product of base and height. True.

2:- Find area of the following:-

(i)



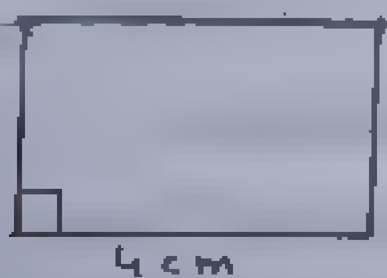
As we know that,

Area of rectangle = Length x Width

" " " = (6) (3)

" " " = 18 cm^2

(ii)



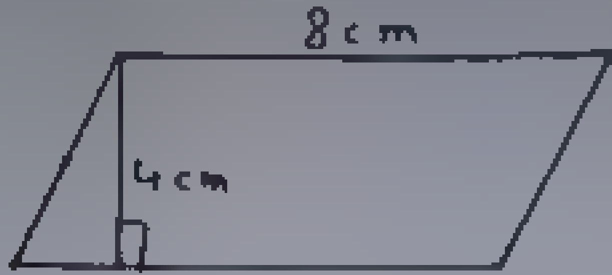
As we know that,

Area of square = side x side

" " " = 4 x 4

" " " = 16 cm^2

(iv)



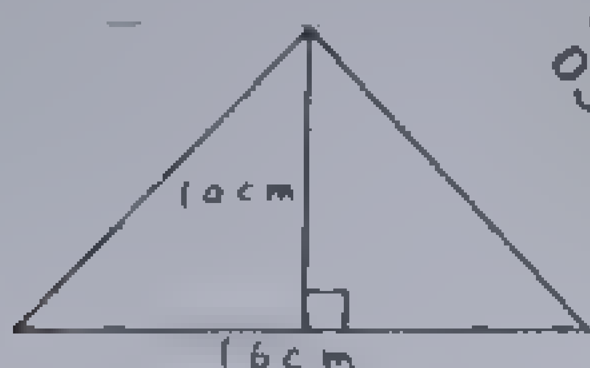
As we know that,

Area of parallelogram = Base × Altitud

" " " $= 8 \times 4$

" " " $= 32\,cm^2$

(v)



As we know that,

Area of triangle $= \frac{1}{2} \times$ Base × Altitude

" " " $= \frac{1}{\cancel{2}_1} \times \cancel{16}^{8} \times 10$

" " " $= 80\,cm^2$

Chapter No 17:-
"Practical Geometry — Triangles"

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Basic Concepts

i) Angle Bisector.
ii) Perpendicular Bisector.
iii) Median.
iv) Altitude.
v) Concurrent lines & Point of concurrency
vi) Centroid.
vii) Incentre.

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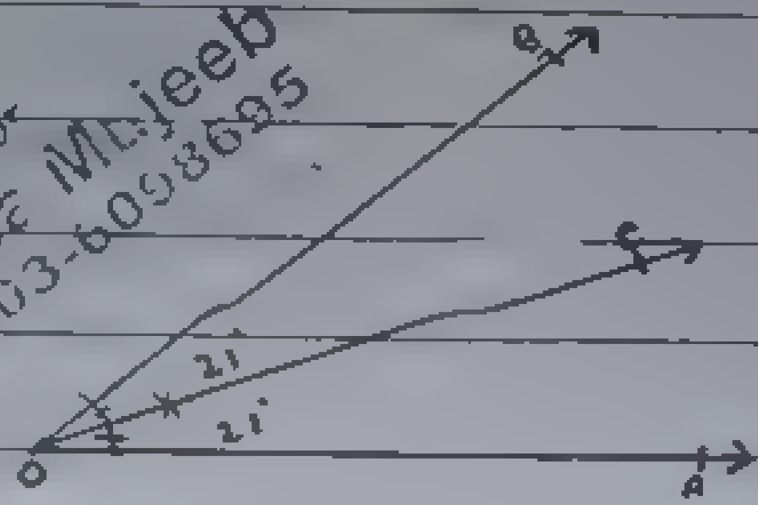
viii) Orthocentre.
ix) Circum-Centre.
x) Observe that.
xi) Ex 17.1 (complete).
xii) Ex 17.2 (complete).
xiii) Ex 17.3 (Q1 (i) & iii Only).
xiv) Review Ex 17.

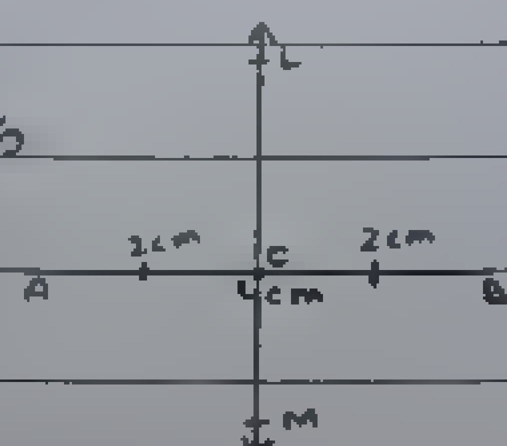
i) Angle Bisector:-

Angle bisector is the ray which divides an angle into two equal parts.



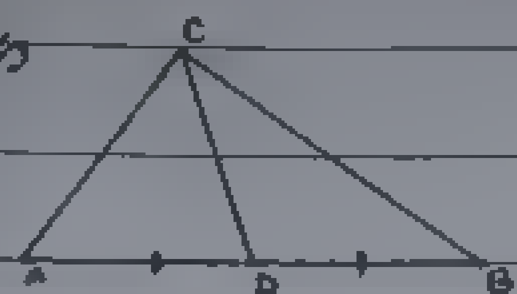
ii) Perpendicular Bisector:-

A line l is called perpendicular bisector of line segment if l is perpendicular to the line segment and passes through its mid-point.



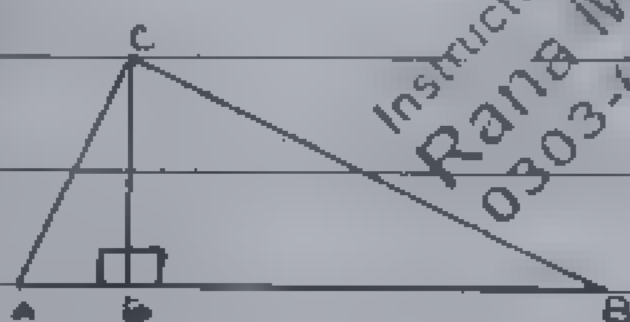
iii) Median:-

A line segment joining a vertex of a triangle to the mid-point of the opposite side is called median of a triangle.



iv) Altitude:-

A line segment from a vertex of a triangle, perpendicular to the line containig opposite side, is called altitude of the Δ.



v) Concurrent lines & Point of concurrency:-

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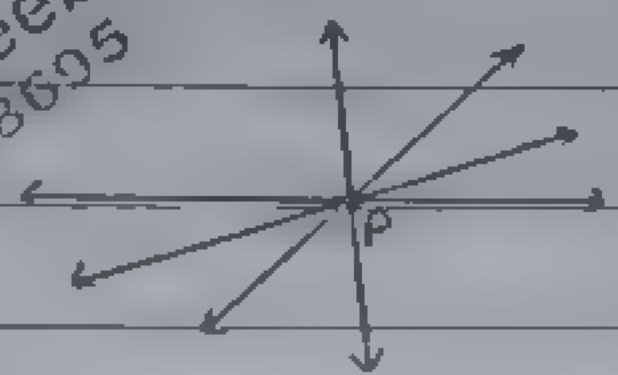
Three or more than three lines are said to be concurrent, if they all pass through the same point. The common point is called point of concurrency.



Here, P is a point of concurrency.

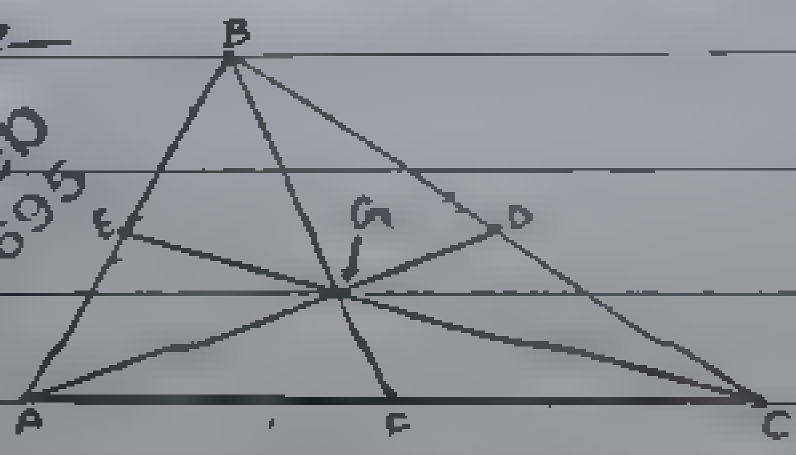
vi) Centroid:-

The point where the three midians of Δ meet is called centroid of Δ.

Location:-

It is always inside the Δ.

Figure:-



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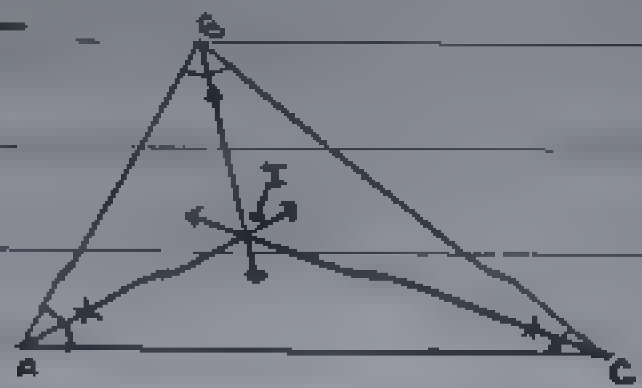
vii) Incentre:-

The internal bisector of the angles of Δ meet at a point called incentre of Δ.

Location:-

It is always inside the Δ.

Figure:-



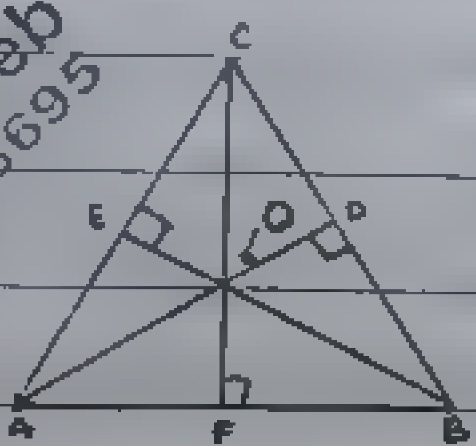
viii) Orthocentre:-

The point of concurrency of the three altitudes of Δ is called its orthocentre.

Location:-

- In Acute Δ, it is inside.
- In Right Δ, it is at Hypotenuse.
- In Obtuse Δ, it is outside.

Figure:-



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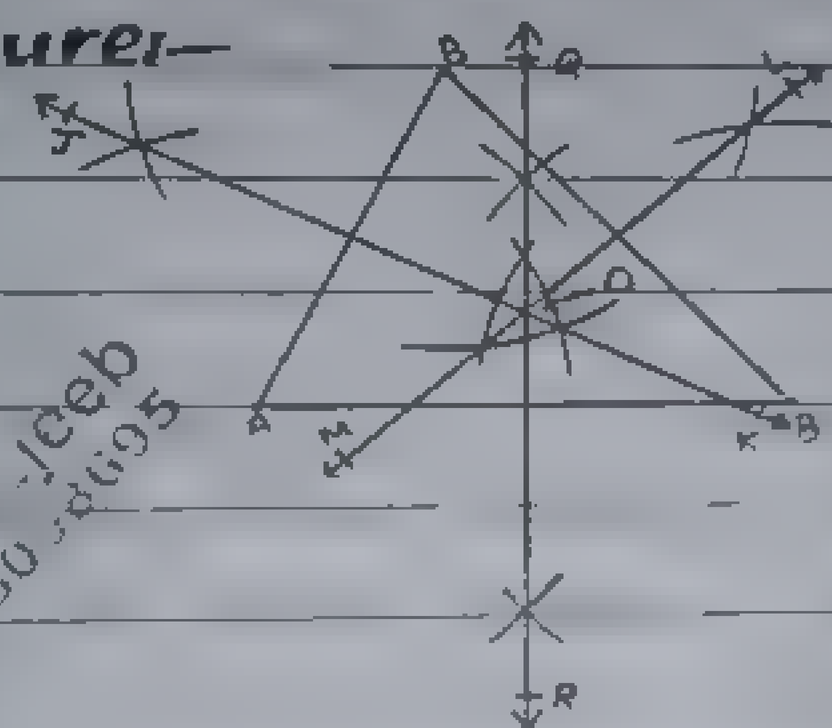
ix) Circumcentre:-

The point of concurrency of the perpendicular bisectors of the sides of a Δ is called circumcentre of Δ.

Location:-

In Acute Δ, it is inside.
In Right Δ, it is at Hypotenuse.
In Obtuse Δ, it is outside.

Figure:-



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x) Observe that:-

Δs APC, ADC stand on the same base AC and b/w the same parallels AC and PD.

Hence ΔAPC = Δ ADC

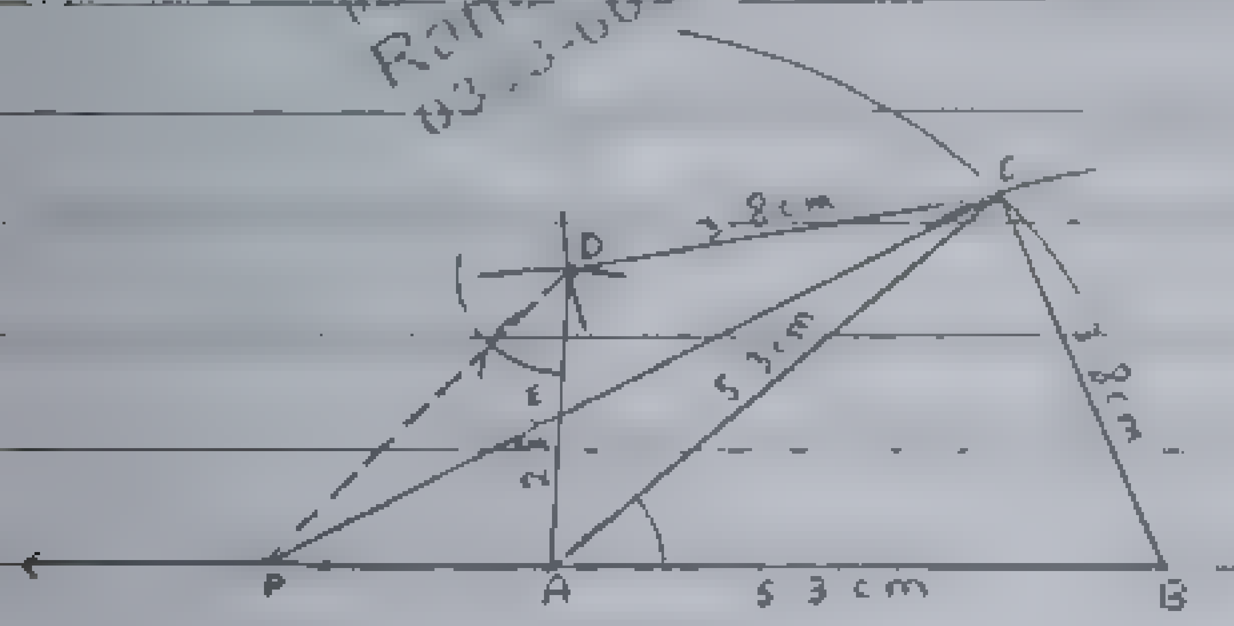
ΔAPC + ΔABC = ΔADC + ΔABC or ΔPBC = quadilateral (ABCD)

xiii) Ex 17.3

1.

(i). Contruct a quidilateral ABCD, having $m\overline{AB} = m\overline{AC} = 5.3\,cm$, $m\overline{BC} = m\overline{CD} = 3.8\,cm$ and $m\overline{AD} = 2.8\,cm$.

(ii). On the side BC construct a Δ equal in area to the quadilateral ABCD.



Construction:-

(i) Draw a line segment $m\overline{AB} = 5.3cm$

(ii) Draw an arc of radius 5.3cm with centre A and draw an arc of radius 3.8cm with centre B.

(iii) These two arcs cut each other at point C. Join C to A & C to B.

(iv) Draw an arc of radius 3.8cm with centre C and daw an arc of radius 2.8cm with centre A.

(v) These two arcs cut each other at point D. Join D to C and D to A to complete quidilateral ABCD.

(vi) Through D, draw $DP \parallel AC$, meeting BA produced at P.

(vii) Join P to C.

Hence ΔPBC is required Δ and its area is equal to quadilateral ABCD.

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xiv) Review Ex 17

1. **Fill in the blanks to make statement true.**

(i) The side of a right angled triangle opposite to 90° is called **Hypotenuse.**

(ii) The line segment joining a vertex of triangle to mid-point of its opposite side is called a **median.**

(iii) A line drawn from a vertex of a triangle which is **Perpendicular** to its opposite side is called an altitude of triangle.

(iv) The bisector of three angles of a triangle are **concurrent**.

(v) The point of concurrency of the right bisectors of the three sides of the triangle is **equidistant** from its vertices.

(vi) Two or more triangles are said to be similar if they are equiangular and measures of their corresponding sides are **Proportional**

(vii) The altitudes of a right triangle are concurrent at the **Vertex** of the right angle.

2. Multiple Choice Questions. Choose the correct answer.

(i) A $\triangle$ having two sides congruent is called ______.

(a) scalene (b) right angled

(c) equilateral (d) isosceles.

(ii) A quadrilateral having each angle of $90°$ is called ______.

(a) parallelogram (b) rectangle

(c) trapezium (d) rhombus

(iii) The right bisectors of the three sides of a triangle are ______.

(a) congruent (b) collinear

(c) concurrent (d) parallel

(iv) The ______ altitudes of an isosceles triangle are congruent.

(a) two (b) three

(c) four (d) none.

(v) A point equidistant from the end points of a line-segement is on its ______.

(a) bisector (b) right bisector

(c) perpendicular (d) median

(vi) ______ congruent triangles can be made by joining the mid-points of the sides of a triangle.

(a) three (b) four

(c) five (d) two

(vii) The diagonal of a parallelogram ______ each other.

(a) bisect (b) trisect

(c) bisect at right angle (d) none of these

(viii) The medians of a triangle cut each other in the ratio ____.

(a) 4:1 (b) 3:1

(c) 2:1 (d) 1:1

(ix) One angle on the base of an isosceles triangle is 30°. What is the measure of its verticle angle ____.

(a) 30° (b) 60°

(c) 90° (d) 120°

(x) If the three altitudes of a triangle are congruent, then the triangle is ____.

(a) equilateral (b) right angled

(c) isosceles (d) acute angled

(xi) If two medians of a triangle are congruent then the triangle is ____.

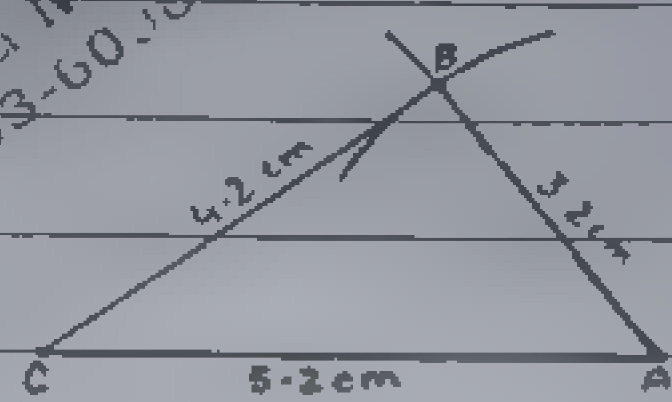
(a) isosceles (b) equilateral

(c) right angled (d) acute angled

xi) Ex 17.1

1. Construct a ΔABC, In which

(i) $m\overline{AB} = 3.2cm$, $m\overline{BC} = 4.2cm$, $m\overline{CA} = 5.2cm$



Construction—

(i) Draw a line segment $m\overline{CA} = 5.2cm$

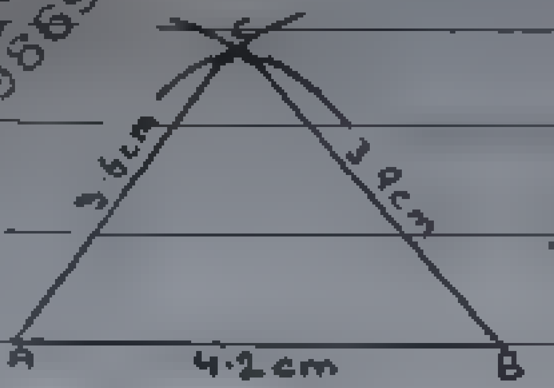
(ii) Draw an arc of radius 4.2cm and draw an arc of radius 3.2 cm.

(iii) These two arcs intersect each other at a point B.

(iv) Join B to C and B to A.

Hence, ΔABC is required Δ.

(ii) $m\overline{AB} = 4.2\,cm$, $m\overline{BC} = 3.9\,cm$, $m\overline{CA} = 3.6\,cm$



Construction:-

(i) Draw a line segment $m\overline{AB} = 4.2\,cm$

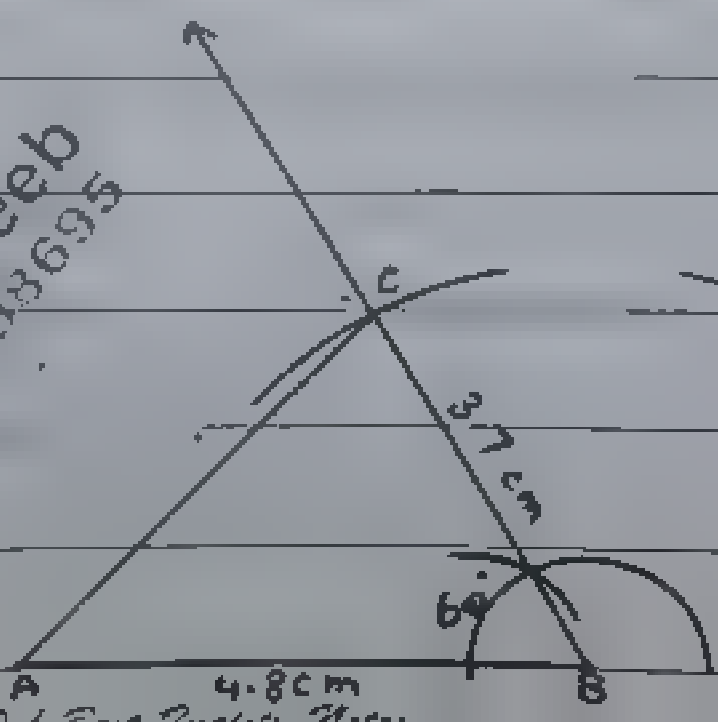
(ii) Draw an arc of radius 3.9cm with centre B and draw an arc of radius 3.6 cm with centre A.

(iii) These two arcs intersect each other at a point C.

(iv) Join C to A & C to B.

Hence, $\triangle ABC$ is required $\triangle$.

(iii) $m\overline{AB} = 4.8\,cm$, $m\overline{BC} = 3.7\,cm$, $m\angle B = 60^\circ$



Construction:-

(i) Draw a line segment $m\overline{AB} = 4.8cm$

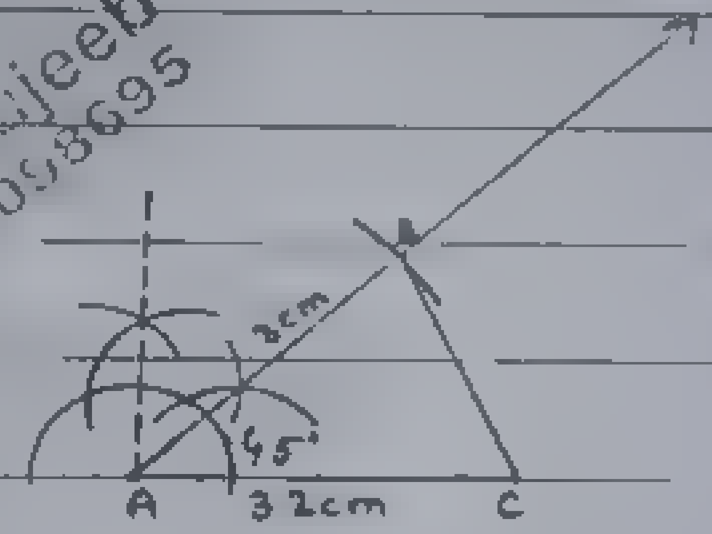
(ii) Make an angle of $m\angle B = 60^\circ$

(iii) Draw an arc of radius 3.7cm with centre B.

(iv) Join C to A.

Hence, ΔABC, is required Δ.

(iv) $m\overline{AB} = 3cm$, $m\overline{AC} = 3.2cm$, $m\angle A = 45^\circ$



Construction:-

(i) Draw a line segment $m\overline{AC} = 3.2cm$

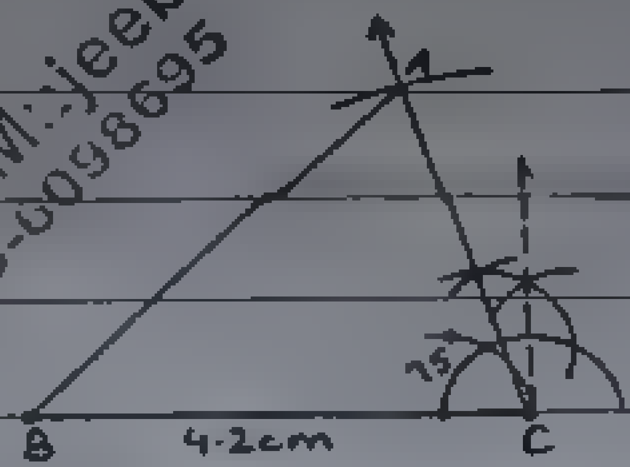
(ii) Make an angle of $m\angle A = 45^\circ$

(iii) Draw an arc of radius 3cm with centre A.

(iv) Join B to C.

Hence, ΔABC is a required Δ.

(v) $m\overline{BC} = 4.2cm$, $m\overline{CA} = 3.5cm$, $m\angle C = 75^\circ$



Construction:-

(i) Draw a line segment $m\overline{BC} = 4.2cm$

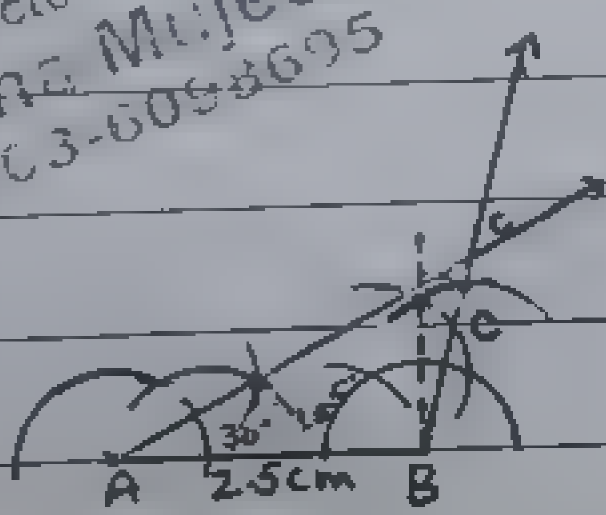
(ii) Make an angle of $m\angle C = 75^\circ$

(iii) Draw an arc of radius 3.5cm with centre C.

(iv) Join A to B.

Hence, ΔABC is required Δ.

(vi) $m\overline{AB} = 2.5cm$, $m\angle A = 30^\circ$, $m\angle B = 105^\circ$



Construction:-

(i) Draw a line segment $m\overline{AB} = 2.5cm$

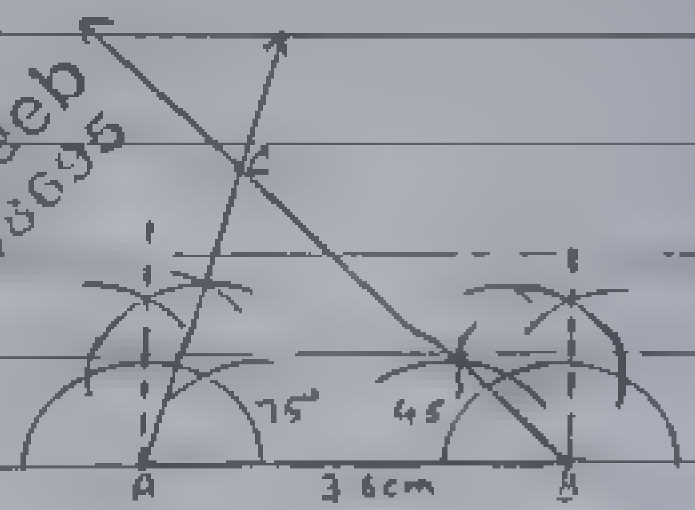
(ii) Make an angle of $m\angle A = 30^\circ$

(iii) Make an angle of $m\angle B = 105^\circ$

(iv) These rays intersect each other at a point C to complete ΔABC.

Hence, ΔABC is a required Δ.

(vii) $m\overline{AB} = 3.6cm, m\angle A = 75^\circ, m\angle B = 45^\circ$



Construction:-

(i) Draw a line segment $m\overline{AB} = 3.6cm$

(ii) Make an angle of $m\angle A = 75^\circ$

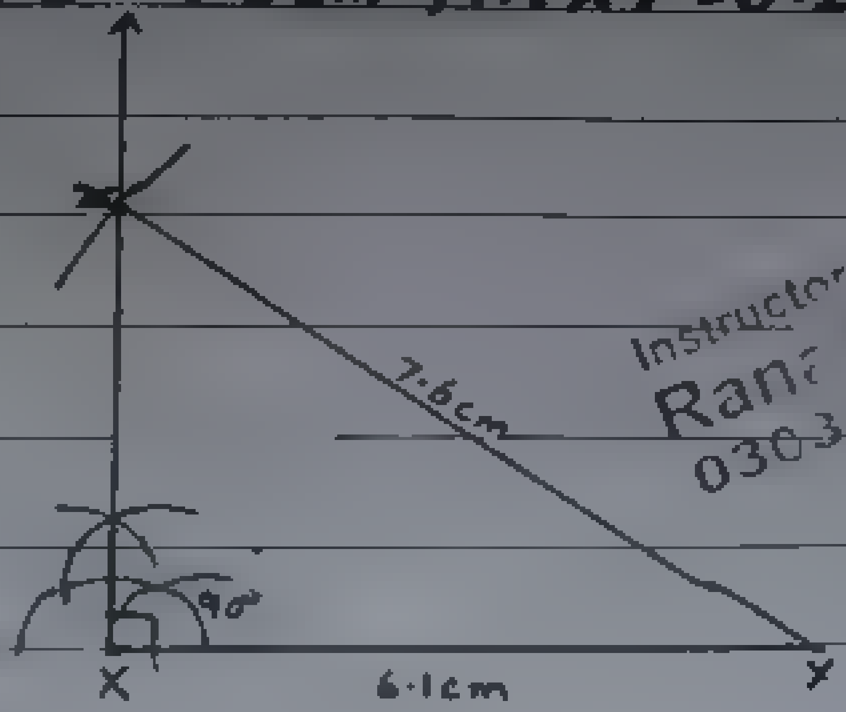
(iii) Make an angle of $m\angle B = 45^\circ$

(iv) These ray intersect each oth at point C to complete ΔABC.

Hence, ΔABC is required Δ

2.- Construct ΔXYZ, in which

(i) $m\overline{YZ} = 7.6cm$, $m\overline{XY} = 6.1cm$, $m\angle X = 90^\circ$



Construction:-

(i) Draw a line segment $m\overline{XY} = 6.1cm$

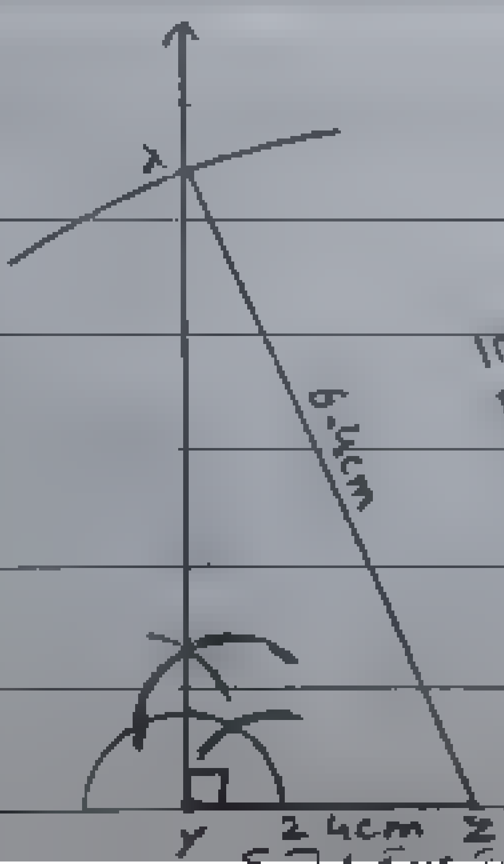
(ii) Make an angle of $m\angle X 90^\circ$.

(iii) Draw an arc of radius $7.6cm$ with centre Y.

(iv) Join Z to Y.

Hence, ΔXYZ is required Δ.

(ii) $m\overline{ZX} = 6.4cm$, $m\overline{YZ} = 2.4cm$, $m\angle Y = 90^\circ$



Construction:-

(i) Draw a line segement $m\overline{YZ} = 2.4cm$

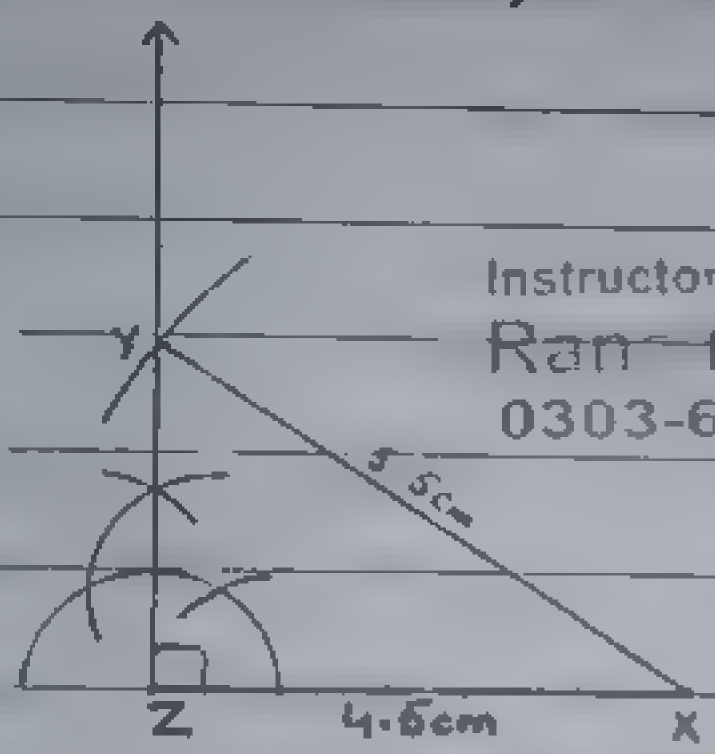
(ii) Make an angle of $m\angle Y = 90^\circ$

(iii) Draw an arc of radius $6.4cm$ with centre Z.

(iv) Join X to Z.

Hence, ΔXYZ is required Δ.

(iii) $m\overline{XY} = 5.5cm$, $m\overline{ZX} = 4.5cm$, $m\angle Z = 90^\circ$



Construction:-

(i) Draw a line segment $m\overline{ZX} = 4.5cm$

(ii) Make an angle of $m\angle Z = 90^\circ$

(iii) Draw an arc of radius $5.5cm$ with centre X.

(iv) Join Y to X.

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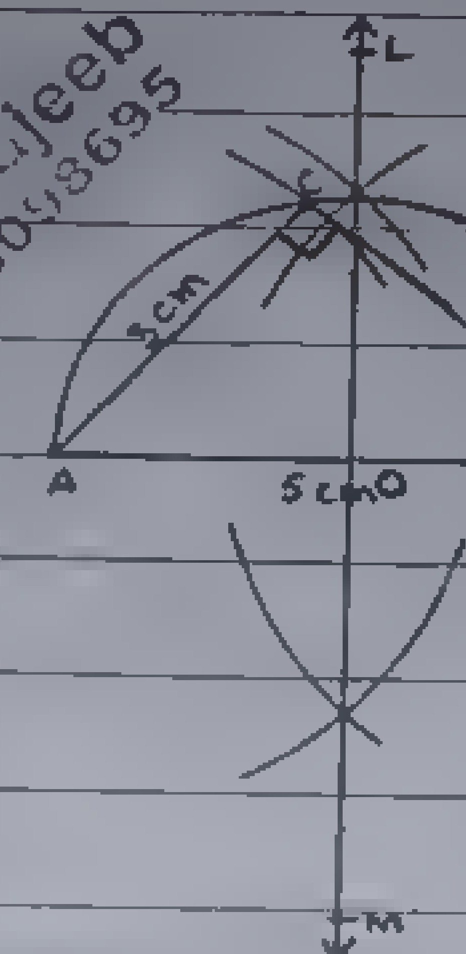
Hence, ΔXYZ is required Δ

3. Construct a right-angled Δ measure of whose hypotenuse is 5cm and one side is 3.2cm.

19



Construction:-

(i) Draw a line segment $m\overline{AB} = 5cm$

(ii) Draw a right Bisector $\overleftrightarrow{LM}$ of $\overline{AB}$ which intersect it at O.

(iii) Draw a semi-circle of radius equals to $m\overline{OA}$ and $m\overline{OB}$.

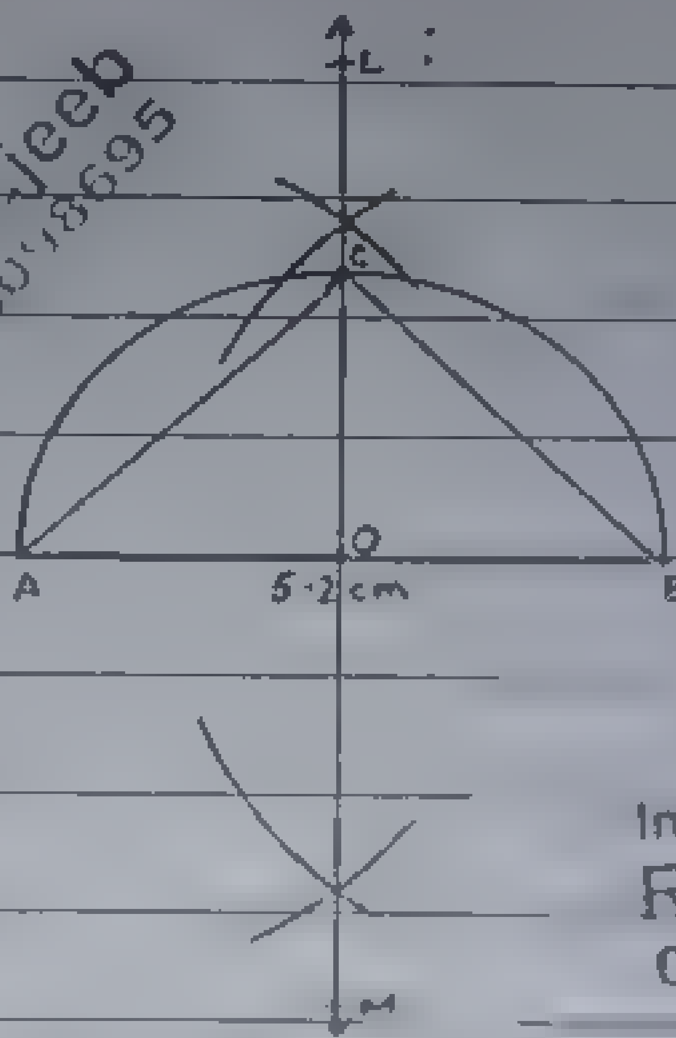
(iv) Draw an arc of radius 3cm from centre A.

(v) Join A to C & B to C

Hence, ΔABC is required Δ.

4. Construct a right-angled isosceles Δ whose hypotenuse is:-

(i) 5.2 cm



Construction:-

(i) Draw a line segment $m\overline{AB} = 5.2$ cm

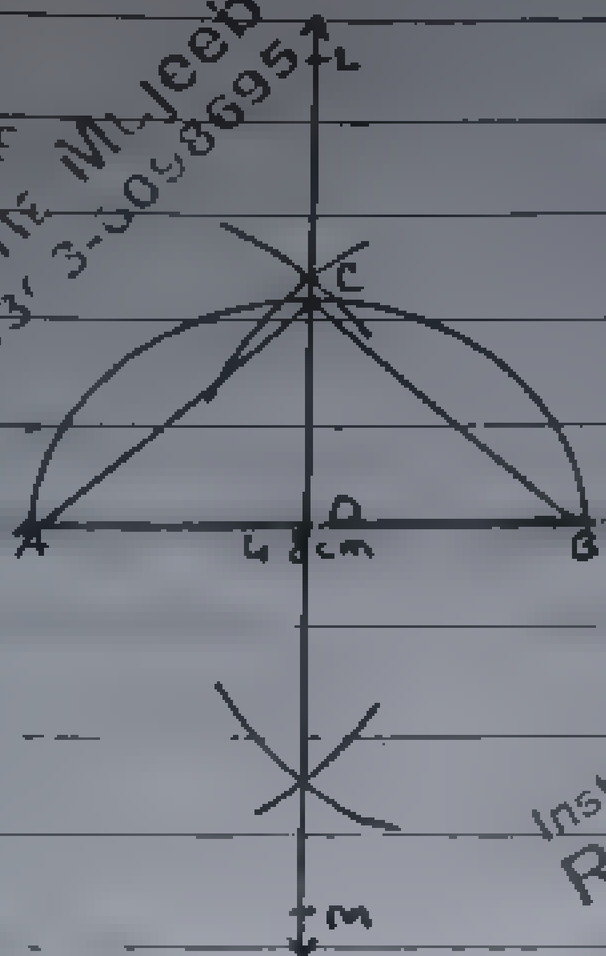
(ii) Draw a right bisector $\overleftrightarrow{LM}$ of $\overline{AB}$ which intersect it at O.

(iii) Draw a semi-circle of radius equals to $m\overline{OA}$ & $m\overline{OB}$.

Join C to A and C to B.

Hence, ΔABC is required Δ

(ii) 4.8 cm



Construction:-

(i) Draw a line segment $m\overline{AB} = 4.8cm$.

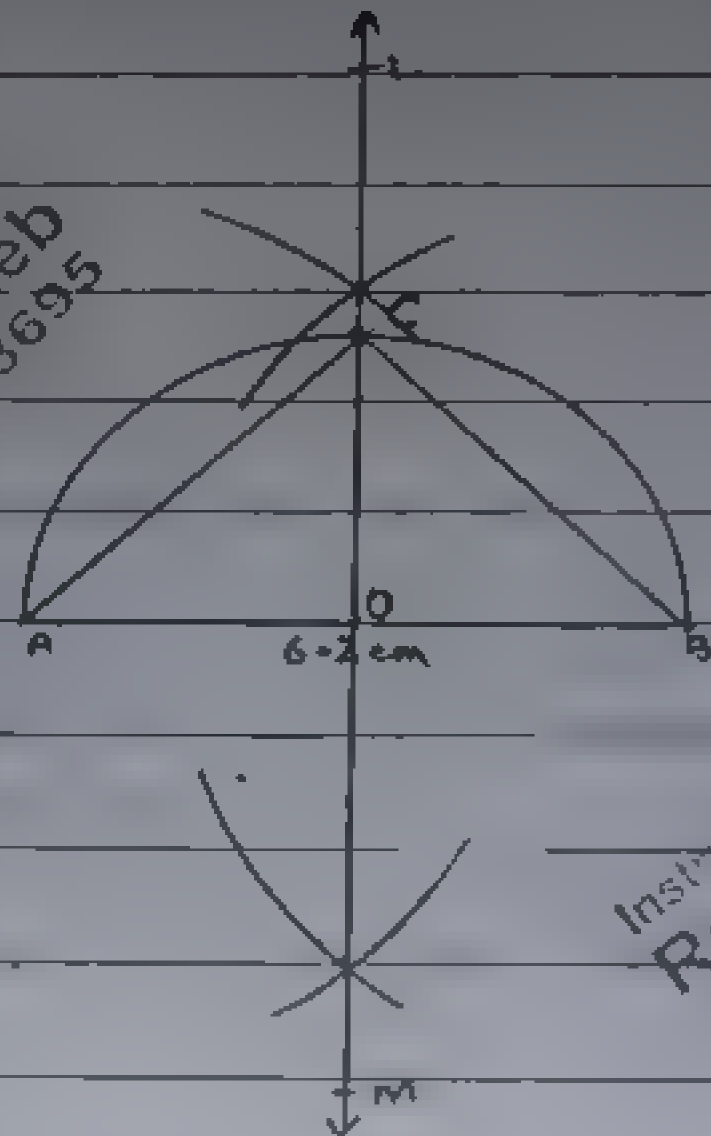
(ii) Draw right bisector of $\overleftrightarrow{LM}$ of $\overline{AB}$ which intersect it at O.

(iii) Draw a semi-circle of radius equals to $m\overline{OA}$ & $m\overline{OB}$

(iv) Join C to A & C to B.

Hence, $\triangle ABC$ is reqiured $\triangle$.

(iii) 6.2cm



Construction:-

(i) Draw a line segment $m\overline{AB} = 6.2cm$

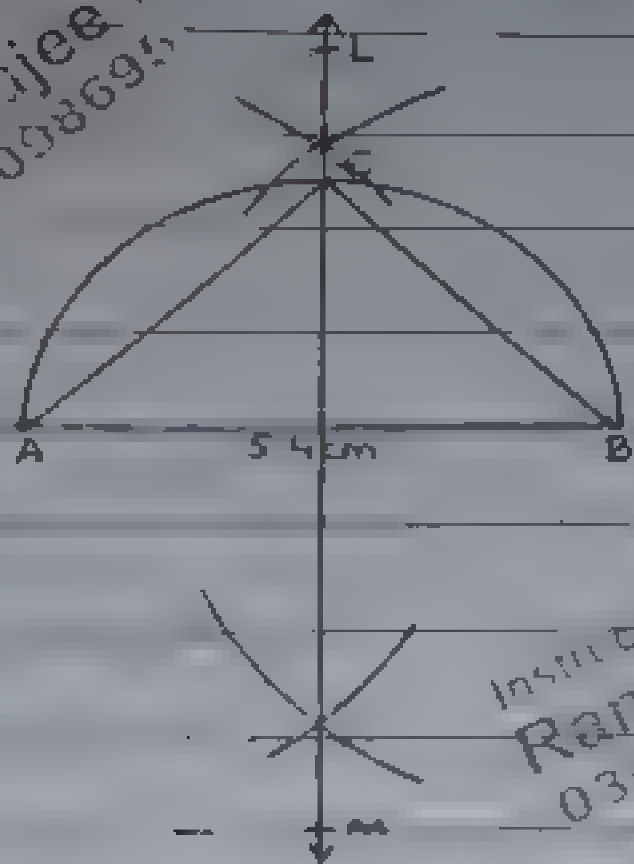
(ii) Draw right bisector $\overleftrightarrow{LM}$ of $\overline{AB}$ which bisect it at O.

(iii) Draw a semi-circle of radius equals to $m\overline{OA}$ & $m\overline{OB}$

(iv) Join C to A and C to B.

Hence, ΔABC is required Δ

(iv) 5.4cm



(i) Draw a line segment $m\overline{AB} = 5.4cm$

(ii) Draw a right bisector $\overleftrightarrow{LM}$ of AB which intersect at it O.

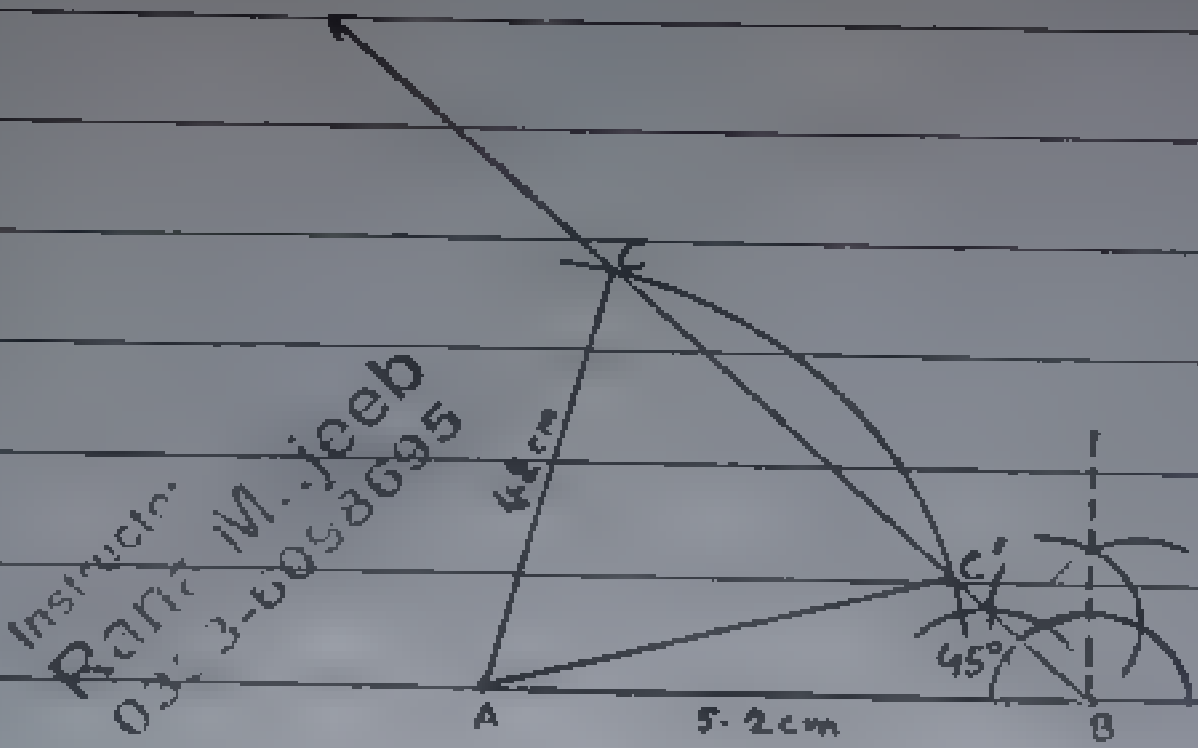
(iii) Draw a semi-circle of radius equals to $m\overline{OA}$ & $m\overline{OB}$

(iv) Join C to A & C to B

Hence, $\triangle ABC$ is required $\triangle$.

5. (Ambiguous Case) Construct a ΔABC, in which:-

(i) $m\overline{AC} = 4.2cm$, $m\overline{AB} = 5.2cm$, $m\angle B = 45^\circ$



Construction:-

(i) Draw a line segment $m\overline{AB} = 5.2cm$

(ii) Make an angle of $m\angle B = 45^\circ$

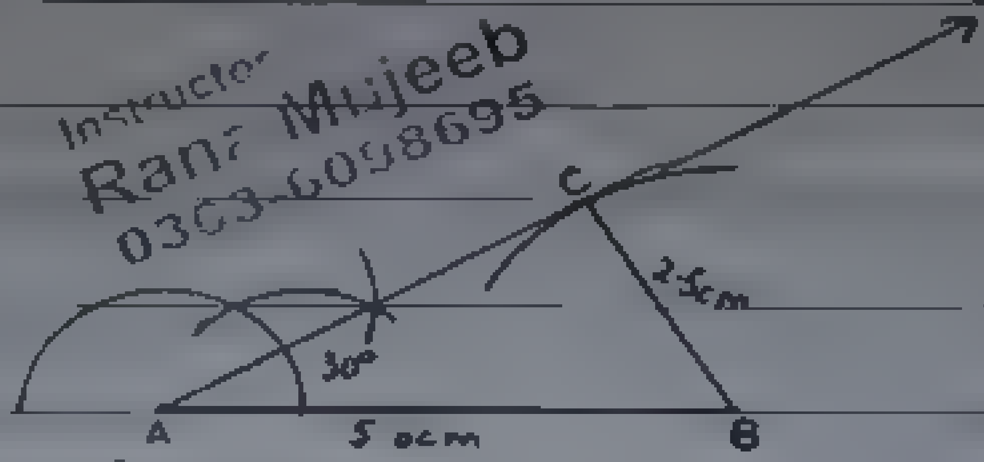
(iii) Draw an arc of radius 4.2cm with centre A.

(iv) The arc intersect the ray at C & C'.

(v) Join A to C & A to C'.

Hence, ΔABC & $\Delta ABC'$ are constructed

(ii) $m\overline{BC} = 2.5cm$, $m\overline{AB} = 5.0cm$, $m\angle A = 30^\circ$



Construction:-

(i) Draw a line segment $m\overline{AB} = 5.0cm$

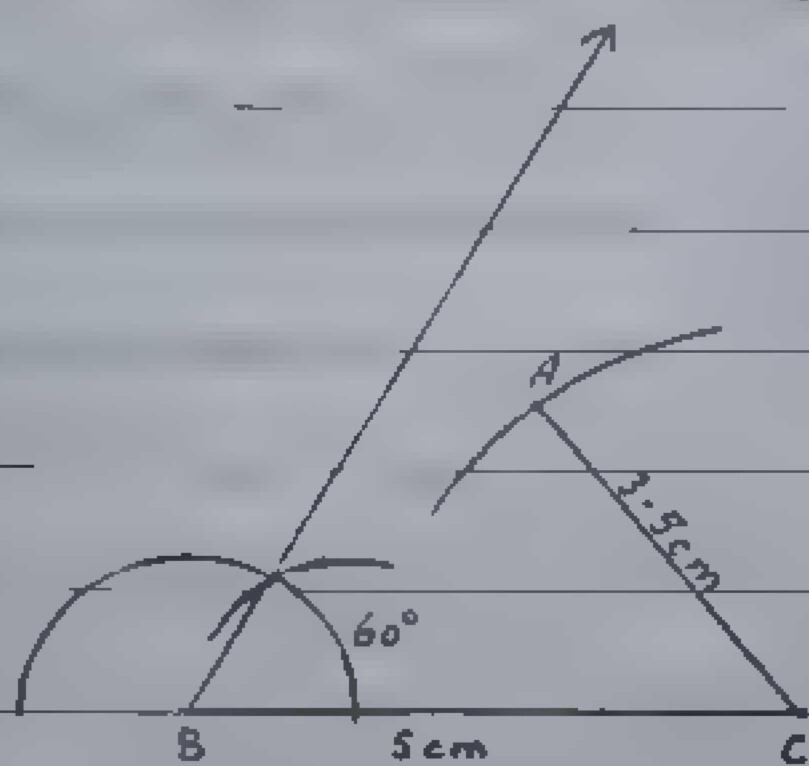
(ii) Make an angle of $m\angle A = 30^\circ$

(iii) Draw an arc of radius 2.5cm with centre B.

(iv) Join B to C.

Hence, $\triangle ABC$ is constructed.

(iii) $m\overline{BC} = 5cm$, $m\overline{AC} = 3.5cm$, $m\angle B = 60^\circ$



Construction

(i) Draw a line segment $m\overline{BC} = 5cm$

(ii) Make an angle of $m\angle B = 60^\circ$

(iii) Draw ar arc of radius 3.5cm with centre C.

(iv) The arc does not intersect the ray at any point.

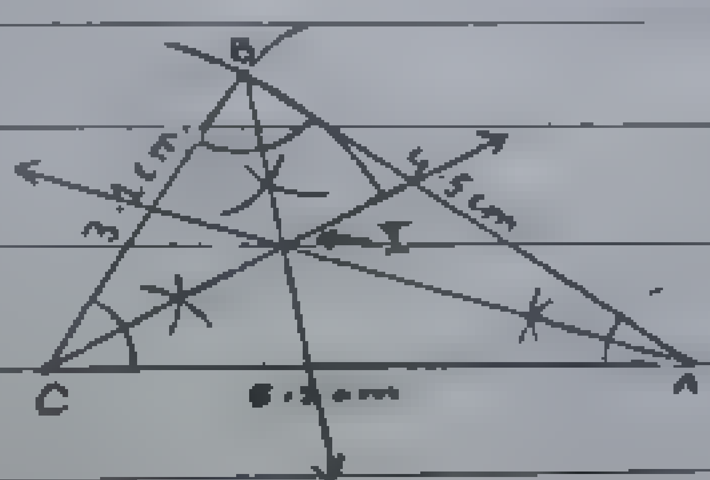
Hence, $\triangle ABC$ cannot be constructed.

xiii)

Ex 17.2

1. Construct the following $\triangle$s ABC. Draw the bisectors of their angles and verify their concurrency.

(i) $m\overline{AB} = 4.5cm$, $m\overline{BC} = 3.1cm$, $m\overline{CA} = 5.2cm$



Construction

(i) Draw a line segment $m\overline{CA} = 5.2cm$

(ii) Draw an arc of radius 4.5cm from centre A.

(iii) Draw an arc of radius 3.1cm from centre C.

(iv) These two arcs cut each other at B.

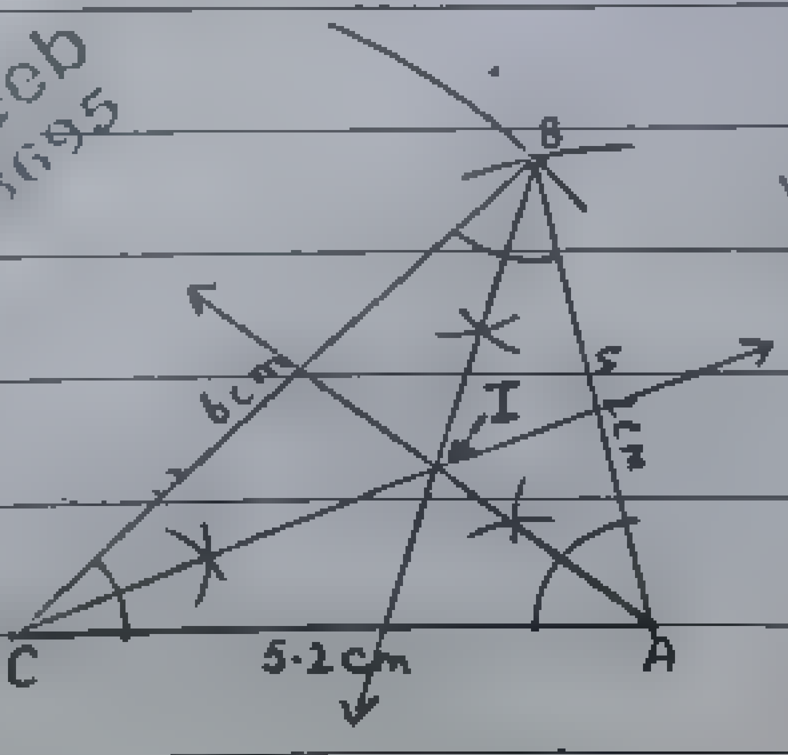
(v) Join A to B and B to C to complete Δ ABC.

(vi) Draw angle bisector of $\angle C$ and $\angle A$ which meets at I.

(vii) Draw angle bisector of $\angle B$ which also meet at I.

Hence, concurrency of angle bisectors of ΔABC is proved.

(ii) $m\overline{AB} = 4.2cm$, $m\overline{BC} = 6cm$, $m\overline{CA} = 5.2cm$



Construction:-

(i) Draw a line segment = 5.2cm

(ii) Draw an arc of radius 6cm with centre C.

(iii) Draw an arc of radius 4.2cm with centre A.

(iv) These B two arce cut each outher at B.

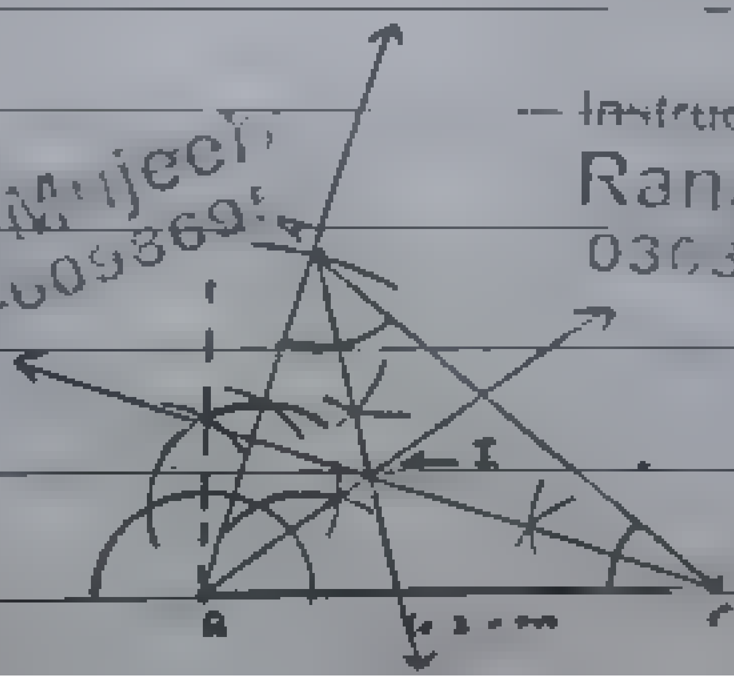
(v) Join B to C & B to A to complete $\triangle ABC$.

(vi) Draw angle bisector of $\angle C$ & $\angle A$ which meets at I.

(vii) Draw angle bisector of $\angle B$ which also meet at I.

Hence, concurrency of angle bisectors of $\triangle ABC$ is proved.

(iii) $m\overline{AB} = 3.6cm, \; m\overline{BC} = 4.2cm, \; m\angle B = 75^\circ$



Construction-

(i) Draw a line segment $m\overline{BC} = 4.2cm$

(ii) Make an angle of $m\angle B = 75^\circ$

(iii) Draw an arc of radius 3.6cm with centre B.

(iv) Join A to C. to complet $\triangle ABC$.

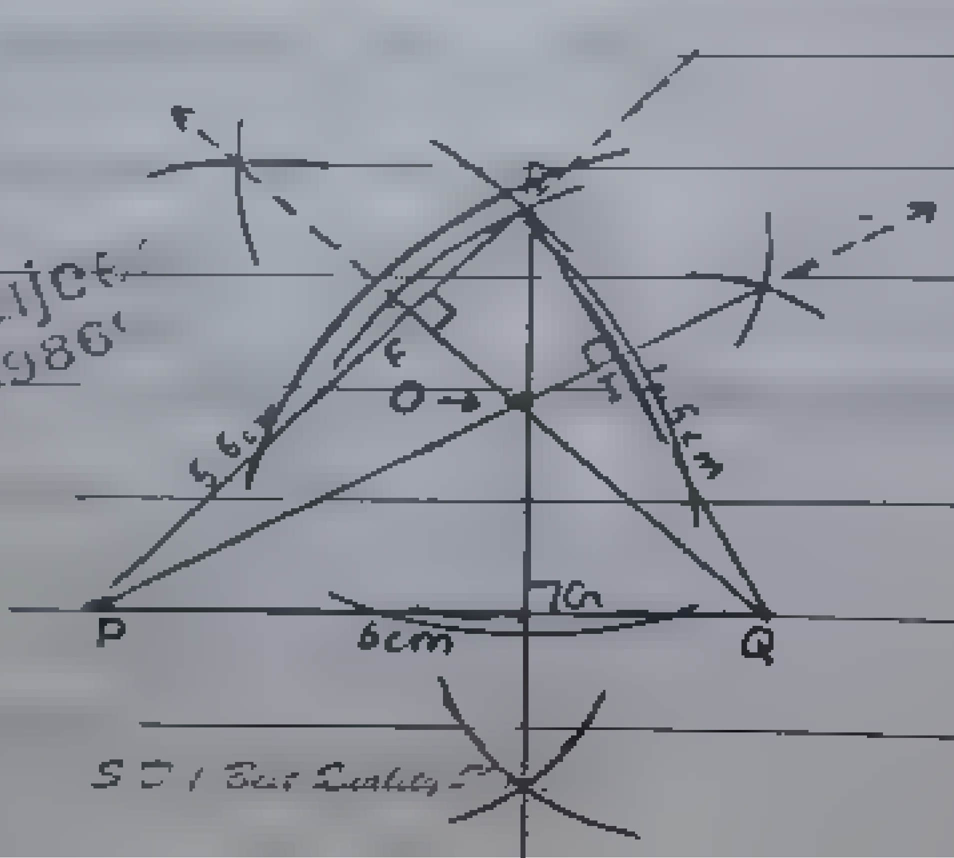
(v) Draw angle bisector of $\angle C$ and $\angle A$ which meets at I.

(vi) Draw angle bisector of $\angle B$ which also meet at I.

Hence, concurrency of angle bisectors of $\triangle ABC$ is proved.

2. Construct the following $\triangle$s PQR. Draw their altitudes and show that they are concurrent.

(i) $m\overline{PQ} = 6cm$, $m\overline{QR} = 4.5cm$, $m\overline{PR} = 5.5cm$



Construction:-

(i) Draw a line segment $m\overline{PQ} = 6cm$

(ii) Draw an arc of radius 5.5cm with centre P. and draw an arc of radius 4.5 cm with centre Q.

(iii) These two arcs cut each other at R.

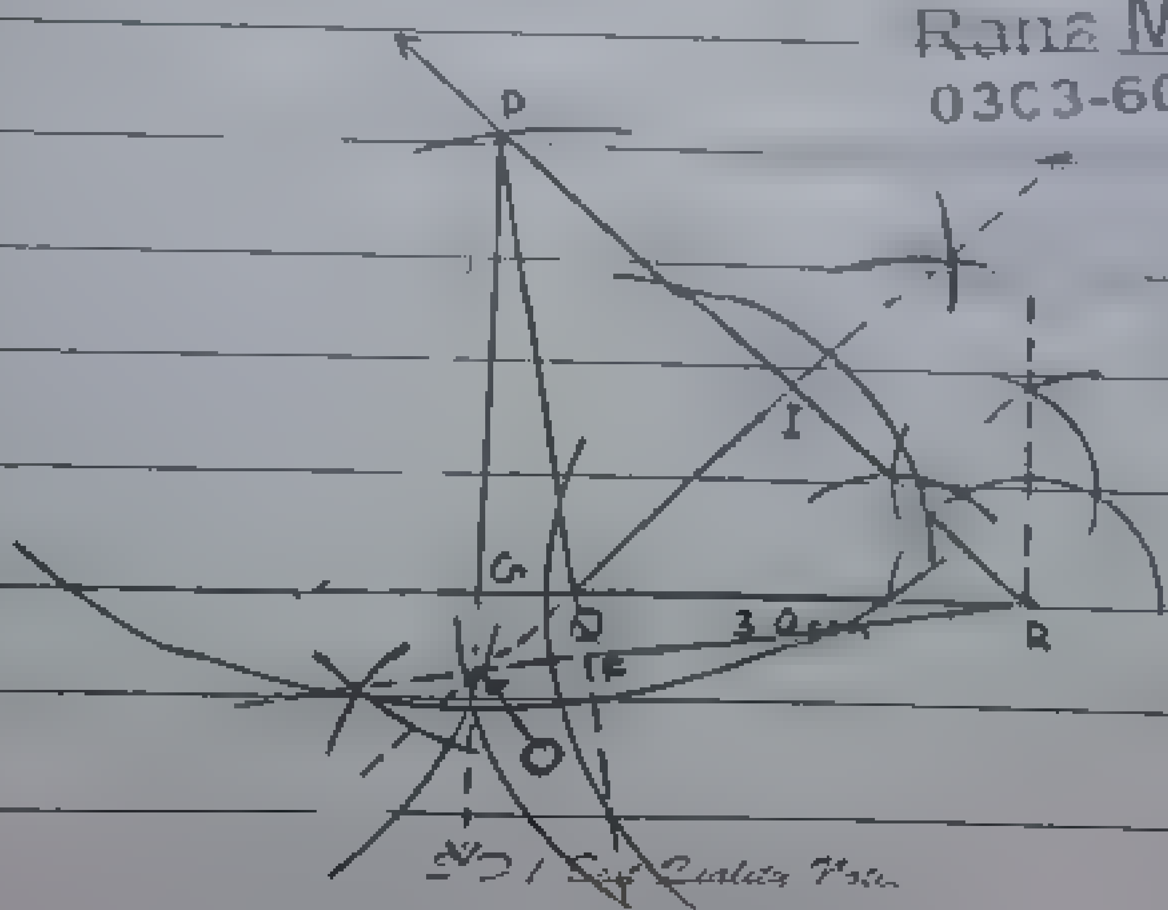
(iv) Join R to P & R to Q to complete ΔPQR

(v) Drop $\overline{IP} \perp \overline{QR}$ and $\overline{QE} \perp \overline{PR}$. These two altitudes meet at O.

(vi) Drop $\overline{GR} \perp \overline{PQ}$ and all altitudes meet at O.

Hence, concurrency of altitudes of ΔPQR is proved.

(iii) $m\overline{PQ} = 4.5cm$, $m\overline{QR} = 3.9cm$, $m\angle R = 45^\circ$



Construction

(i) Draw a line segment $m\overline{QR} = 3.9cm$

(ii) Make an angle of $m\angle R = 45^\circ$

(iii) Draw an arc of radius 4.5cm with centre Q

(iv) Join Q to P. to complete ΔPQR.

(v) Drop $\overline{QI} \perp \overline{PR}$ and $\overline{RF} \perp \overline{PQ}$. These two altitudes meet at O.

(vi) Drop $\overline{PG} \perp \overline{QR}$ and all altitudes meet at O.

Hence, concurrency of altitudes of ΔPQR is proved.

(iii) $m\overline{RP} = 3.6cm$, $m\angle Q = 30^\circ$, $m\angle P = 105^\circ$

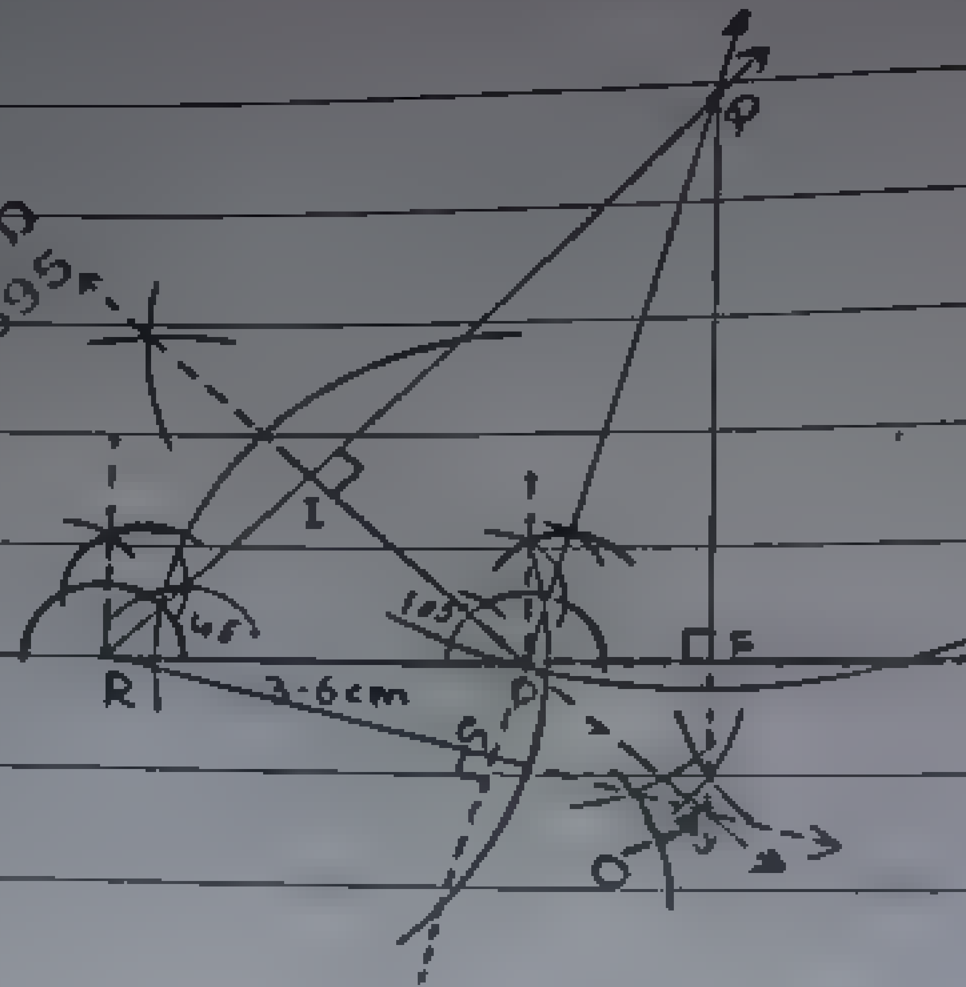
$m\angle R + m\angle Q + m\angle P = 180^\circ$

$m\angle R + 30^\circ + 105^\circ = 180^\circ$

$m\angle R + 135^\circ = 180^\circ$

$m\angle R = 180^\circ - 136^\circ$

$\boxed{m\angle R = 45^\circ}$



Construction:-

(i) Draw a line segment $m\overline{RP} = 3.6cm$

(ii) Make an angles of $m\angle R = 45°$ & $m\angle P = 105°$

(iii) These two rays of $\angle P$ & $\angle R$ cut each other at Q, to complete ΔPQR.

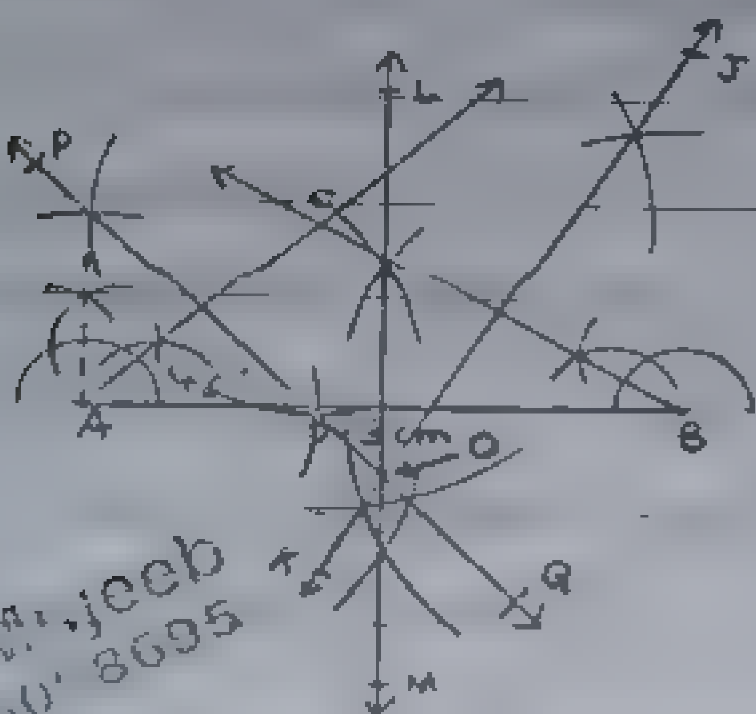
(iv) Drop $\overline{PI} \perp \overline{RQ}$ & $\overline{QF} \perp \overline{RP}$. These two altitudes meet at O.

(v) Drop $\overline{RG} \perp \overline{PQ}$ and all altitudes meet at O.

Hence, concurrency of altitudes of ΔPQR is proved.

3. Construct the following Δs ABC. Draw the perpendicular bisectors of their sides and verify their concurrency. Do they meet inside the Δ?

(i) $m\overline{AB} = 5.3cm,\ m\angle A = 45°,\ m\angle B = 30°$



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Construction:—

(i) Draw a line segment $m\overline{AB} = 5.3cm$

(ii) Make an angles of $m\angle A = 45°$ & $m\angle B = 30°$

(iii) These two rays of $\angle A$ and $\angle B$ cut each other at C. to complete Δ ABC.

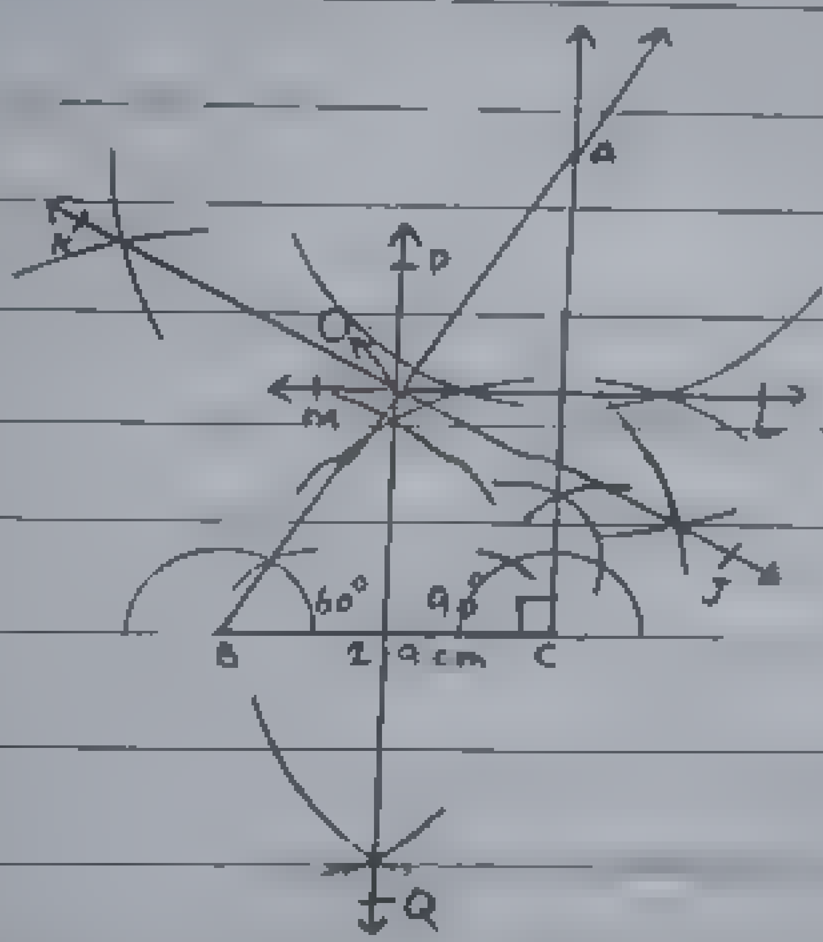
(iv) Draw perpendicular bisector of $\overline{AB}$ and $\overline{BC}$ which meet at O.

(v) Draw perpendicular bisector of $\overline{AC}$ which also meet at O.

Hence, perpendicular bisectors of the sides of $\triangle ABC$ are concurrent at O, which is outside

(ii) $m\overline{BC} = 2.9cm, m\angle A = 30°, m\angle B = 60°$

$$m\angle A + m\angle B + m\angle C = 180°$$
$$30° + 60° + m\angle C = 180°$$
$$90° + m\angle C = 180°$$
$$m\angle C = 180° - 90°$$
$$\boxed{m\angle C = 90°}$$



Construction:-

(i) Draw a line segment $m\overline{BC} = 2.9cm$

(ii) Make an angles of $m\angle B = 60°$ & $m\angle C = 90°$.

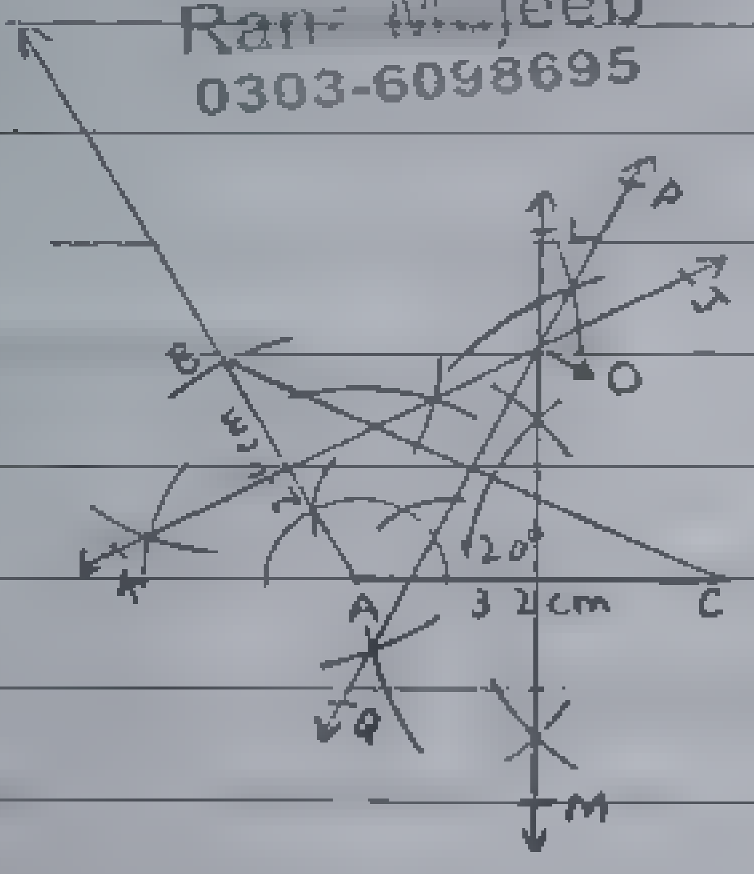
(iii) These two rays of $\angle B$ and $\angle C$ cut each other at A to complete $\triangle ABC$.

(iv) Draw perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$ which meets at O.

(v) Draw perpendicular bisector of $\overline{AC}$ which also meet at O.

Hence, perpendicular bisectors of the sides of $\triangle ABC$ are concurrent at O, which is at hypotenuse of the right $\triangle ABC$.

(iii) $m\overline{AB} = 2.4cm$, $m\overline{AC} = 3.2cm$, $m\angle A = 120^\circ$



Construction:-

(i) Draw a line segment $m\overline{AC} = 3.2cm$

(ii) Make an angle of $m\angle A = 120^\circ$

(iii) Draw an arc of radius 2.4cm with centre A.

(iv) Join A to C to complete $\triangle ABC$.

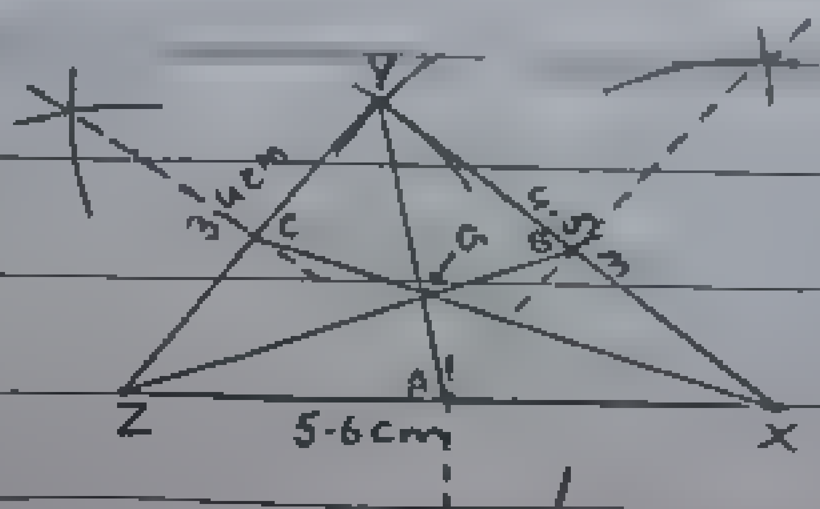
(v) Draw perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$ which meets at O.

(vi) Draw perpendicular bisector of $\overline{AC}$ which also meets at O.

Hence, Perpendicular bisector of the sides of $\triangle ABC$ are concurrent at O, which is outside.

4. Construct the following $\triangle$s XYZ, Draw their three medians and show that they are concurrent:-

(ii) $m\overline{XY} = 4.5cm, m\overline{YZ} = 3.4cm, m\overline{ZX} = 5.6cm$



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Construction

(i) Draw a line segment $m\overline{ZX} = 5.6cm$

(ii) Draw an arc of radius 4.5cm with centre X & draw an arc of radius 3.4cm with centre Z.

(iii) These two arcs cut each other at Y.

(iv) Join Y to Z and Y to X to complete ΔXYZ.

(v) Draw perpendicular bisectors of $\overline{ZX}$, $\overline{YX}$ and $\overline{ZY}$ to find the midpoints as A, B, and C respectively.

(vi) Join Y to A to get median $\overline{YA}$.

(vii) Join Z to B to get median $\overline{ZB}$.

(viii) Join X to C to get median $\overline{XC}$.

Hence all the medians are concurrent at G.

(i) $m\overline{YZ} = 4.1cm$, $m\angle Y = 60^\circ$, $m\angle X = 75^\circ$

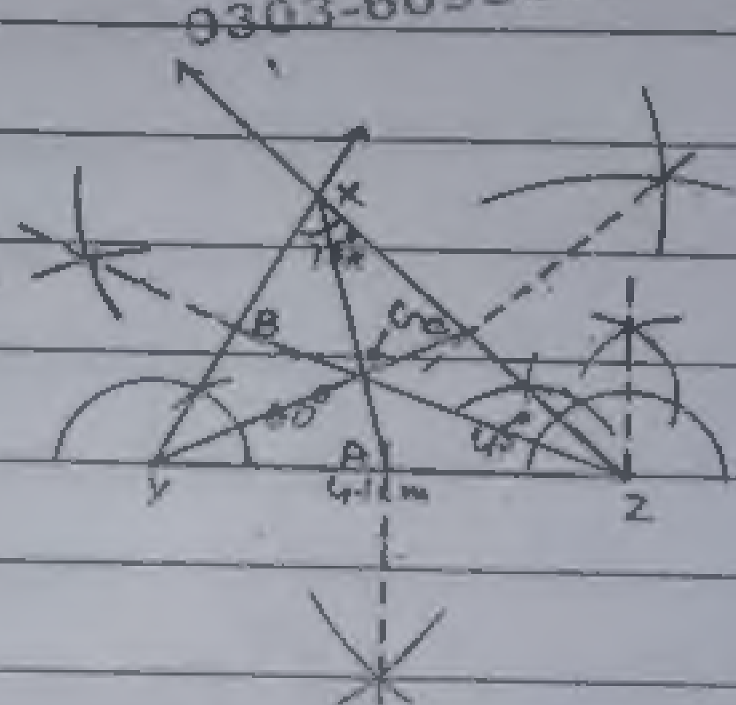
$$m\angle Y + m\angle X + m\angle Z = 180^\circ$$

$$60^\circ + 75 + m\angle Z = 180^\circ$$

$$135^\circ + m\angle Z = 180^\circ$$

$$m\angle Z = 180^\circ - 135^\circ$$

$$\boxed{m\angle Z = 45^\circ}$$



Construction:-

(i) Draw a line segment $m\overline{YZ} = 4.1cm$

(ii) Make an angles of $m\angle X = 60^\circ$ & $m\angle Z = 45^\circ$

(iii) The two rays $\angle X$ and $\angle Z$ cut each other at X to complete $\triangle XYZ$.

(iv) Draw the perpendicular bisectors of $\overline{YZ}$, $\overline{XZ}$, $\overline{XY}$ to find their midpoints A, B and C respectively.

(v) Join X to A to get median $\overline{XA}$.

(vi) Join Y to C to get median $\overline{XC}$.

(vii) Join Z to B to get median $\overline{ZB}$.

Hence, all the medians are concurrent at G.

(iii) $m\overline{ZX} = 4.3cm, m\angle X = 75^\circ, m\angle Y = 45^\circ$

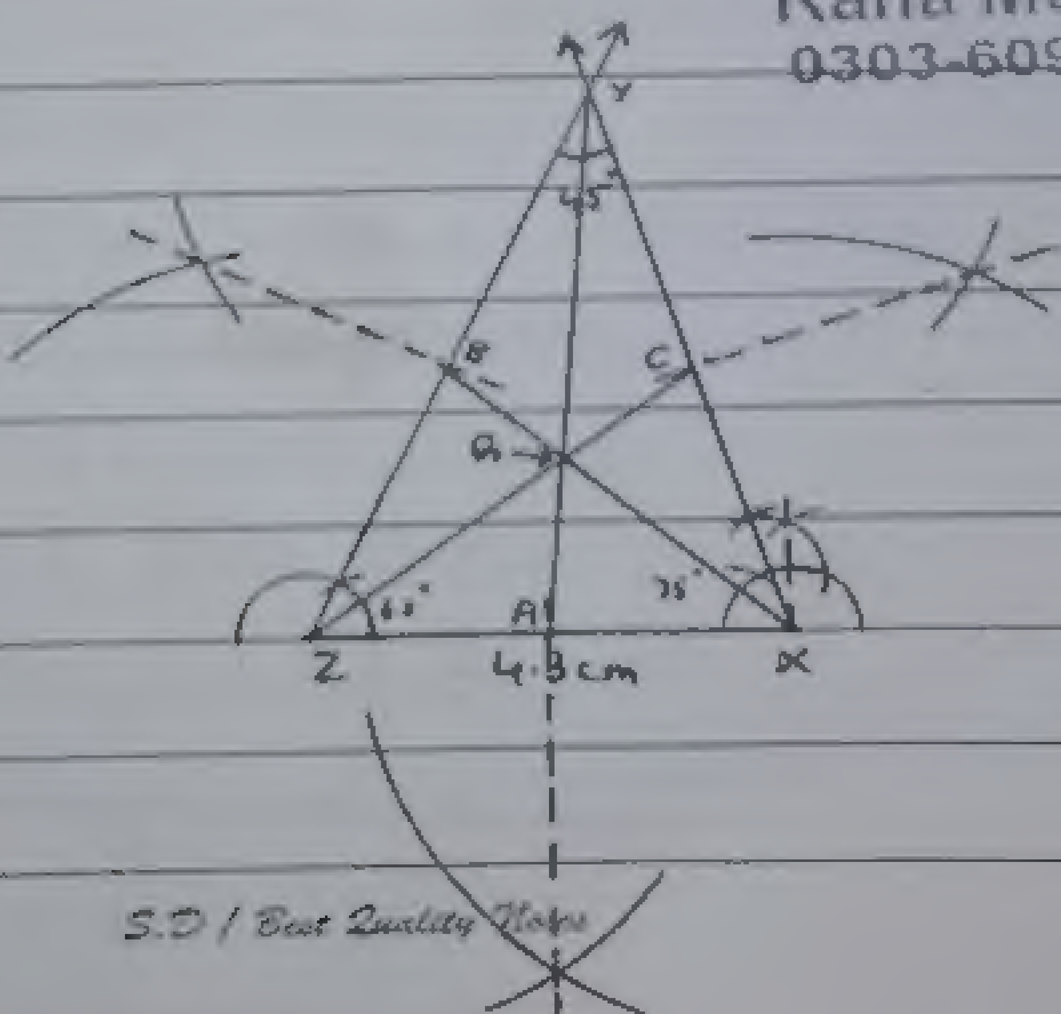
$$m\angle X + m\angle Y + m\angle Z = 180^\circ$$

$$75^\circ + 45^\circ + m\angle Z = 180^\circ$$

$$120^\circ + m\angle Z = 180^\circ$$

$$m\angle Z = 180^\circ - 120^\circ$$

$$\boxed{m\angle Z = 60^\circ}$$



(i) Draw a line segment $m\overline{ZX} = 4.3cm$

(ii) Make an angles of $m\angle Z = 60^\circ$ & $m\angle X = 75^\circ$.

(iii) The two rays $\angle Z$ and $\angle X$ cut each other at Y.

(iv) Draw perpendicular bisectors of $\overline{ZX}$, $\overline{YZ}$ and $\overline{XY}$ to find their mid-point A, B & C respectively.

(v) Join Z to C to get median $\overline{ZC}$.

(vi) Join X to B to get median $\overline{XB}$.

(vii) Join Y to A to get median $\overline{YA}$.

Hence, all the medians are concurrent at G.